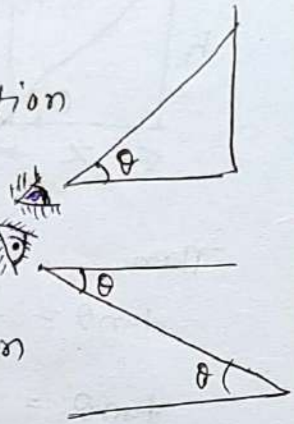
Hight and Distance

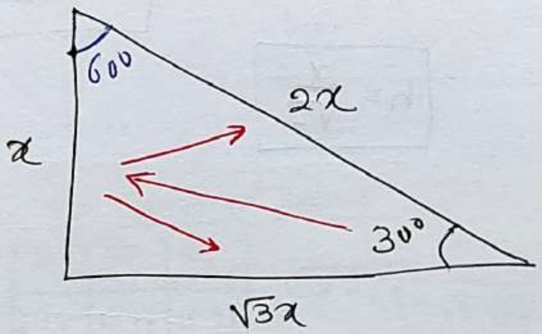
Formulas:-

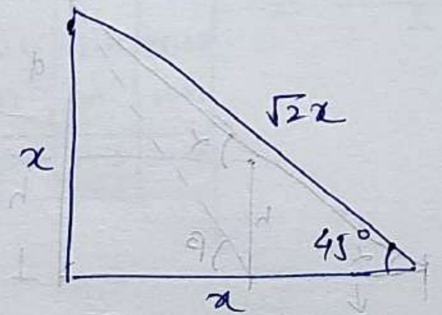
1) θ = Angle of elevation



θ = Angle of depression

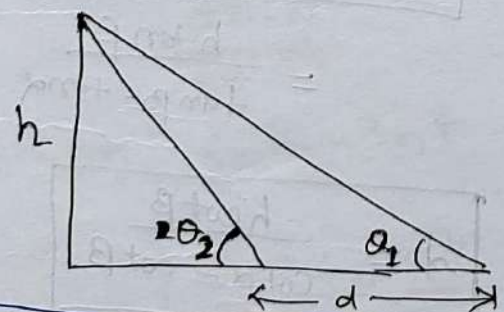
2)



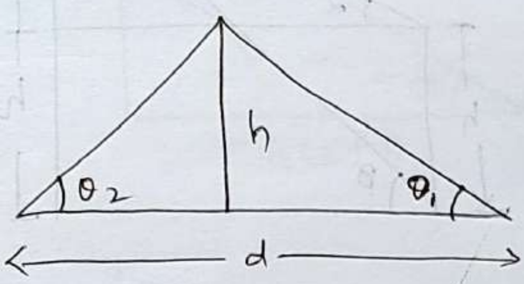


$x \; : \; x \; : \; \sqrt{2}x$

3)

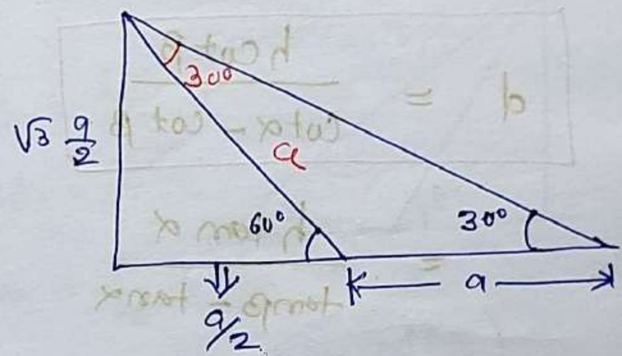


$$\boxed{h = \frac{d}{\cot\theta_1 - \cot\theta_2}}$$



$$\boxed{h = \frac{d}{\cot\theta_1 + \cot\theta_2}}$$

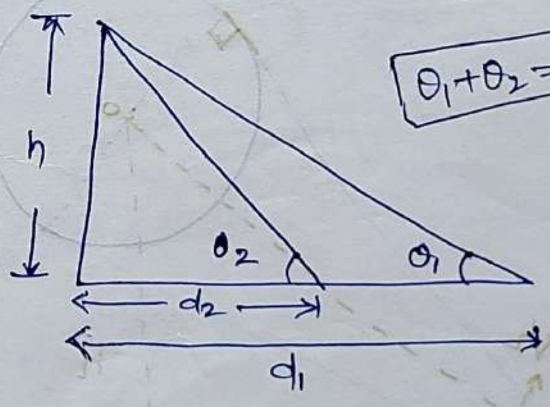
Special case :-



4) $\theta_1 + \theta_2 = 90^\circ$ (Complementary)

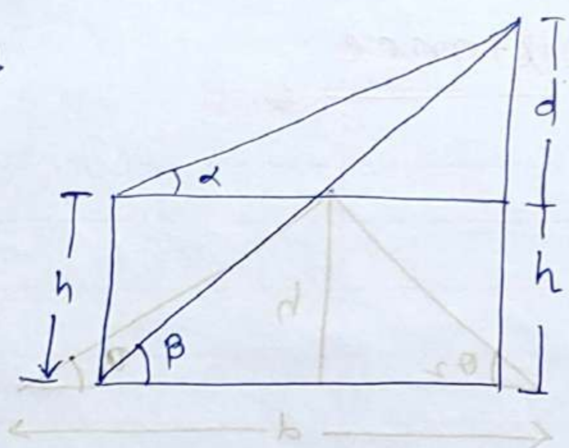
$\Downarrow$

$\boxed{\theta_1 + \theta_2 = 90^\circ}$



$$\boxed{h = \sqrt{d_1 d_2}}$$

6)



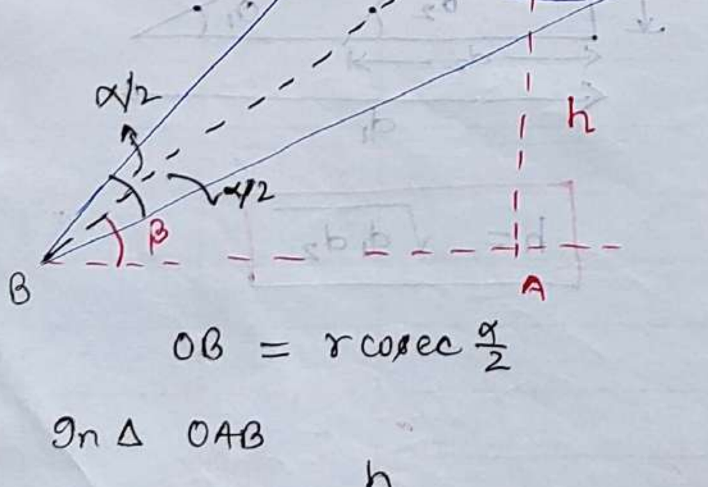
$$\boxed{h+d = \frac{h\cot\alpha}{\cot\alpha-\cot\beta}}$$

Or, $$= \frac{h\tan\beta}{\tan\beta-\tan\alpha}$$

$$\boxed{d = \frac{h\cot\beta}{\cot\alpha-\cot\beta}}$$

Or, $$= \frac{h\tan\alpha}{\tan\beta-\tan\alpha}$$

7)

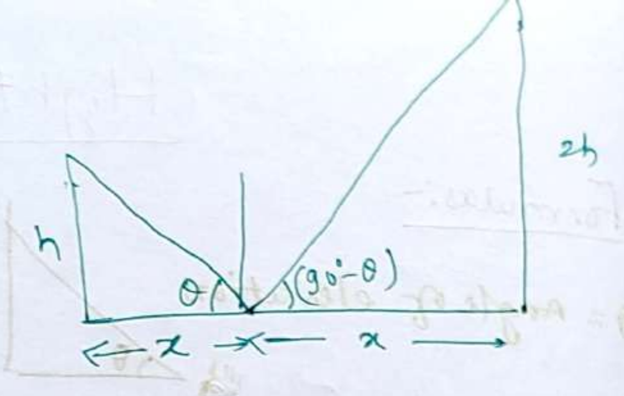


$$OB = r\,\text{cosec}\,\frac{\alpha}{2}$$

In Δ OAB

$$\sin\beta = \frac{h}{OB}$$

$$\boxed{h = OB\sin\beta}$$



Then,

$$\tan\theta = \frac{h}{x}$$

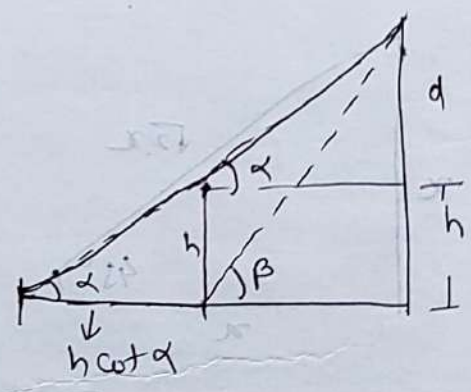
$$\tan\theta = \frac{x}{2h}$$

$$\frac{h}{x} = \frac{x}{2h}$$

$$2h^2 = x^2$$

$$\boxed{h = \frac{x}{\sqrt{2}}}$$

$$\boxed{h = \frac{x}{\sqrt{2}}}$$



$$\boxed{(d+h) = \frac{h\cot\alpha}{\cot\alpha-\cot\beta}}$$

$$= \frac{h\tan\beta}{\tan\beta-\tan\alpha}$$

$$\boxed{d = \frac{h\cot\beta}{\cot\alpha-\cot\beta}}$$

$$d\cot\alpha = (d+h)\cot\beta$$

$$d(\cot\alpha - \cot\beta) = h\cot\beta$$

$$\boxed{d = \frac{h\cot\beta}{\cot\alpha - \cot\beta}}$$

$$(h+d) = \frac{d\cot\alpha}{\cot\beta}$$

$$= \frac{h\cot\alpha\,\cancel{\cot\beta}}{\cancel{\cot\beta}(\cot\alpha - \cot\beta)}$$

$$= \frac{h\cot\alpha}{\cot\alpha - \cot\beta}$$

Compound Interest

%	I	II	III	IV
4%	4	8.16	12.48	16.98
5%	5%	10.25	15.76	21.55
8%	8%	16.64	25.97	36.04
10%	10	21	33.1	46.41
12%	12	25.44	40.49	57.35

$$\sqrt{a - \sqrt{a + \sqrt{a - \sqrt{a}}}} - \infty$$

$$= \frac{\sqrt{4a-3}+1}{2}$$

Compound Interest

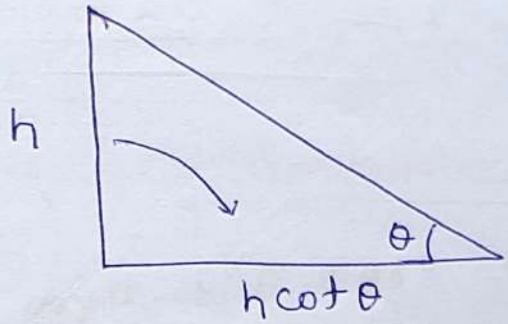
$$3yr = 3r \mid 3r^2 \cdot r^3$$

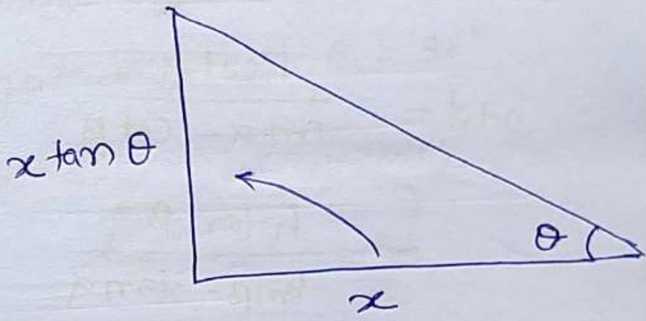
1 2 3 4

$$4yr = 4r \mid 6r^2\, 4r^3\, r^4$$

$$4r \mid 6r^2\, 4r^3\, r^4$$

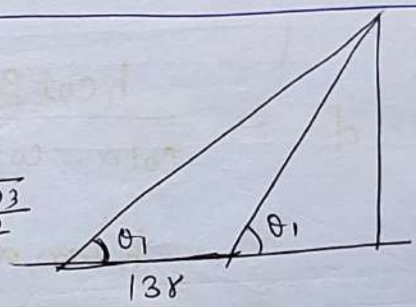
Note:-





News:-

Q



$$\tan\theta_1 = \frac{1}{5}$$

$$\sec\theta_2 = \frac{\sqrt{193}}{12}$$

$$\downarrow$$

$$\tan\theta_2 = \frac{7}{12}$$

$$138 = h(\cot\theta_1 - \cot\theta_2)$$

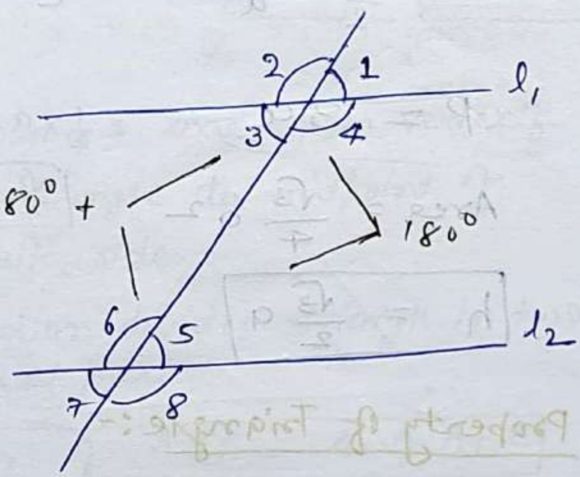
$$h = \frac{138}{5 - \frac{12}{7}} = \frac{138\times 7}{23}$$

$$\boxed{h = 42}$$

GEOMETRY

Straight line | Triangle | Circle | Quadrilateral

Straight line :-



Alternate angle :-

$\angle 3 = \angle 5$] Interial
$\angle 4 = \angle 6$

$\angle 1 = \angle 7$] exterial
$\angle 2 = \angle 8$

vertically opposite :-

$\angle 2 = \angle 4$
$\angle 3 = \angle 1$
$\angle 6 = \angle 8$
$\angle 5 = \angle 7$

Corresponding angles :-

$\angle 1 = \angle 5$
$\angle 2 = \angle 6$
$\angle 3 = \angle 7$
$\angle 4 = \angle 8$

Acute angle : $\theta < 90^\circ$

Obtuse angle : $\theta > 90^\circ - 180$

Right angle : $\theta = 90^\circ$

Supplementary angle :-

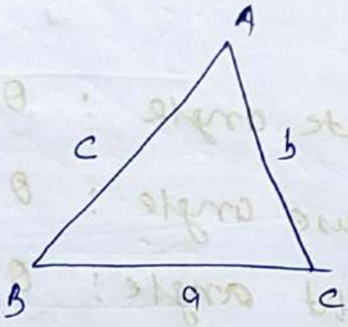
$$\boxed{\theta_1 + \theta_2 = 180^\circ}$$

Complementary angle :-

$$\boxed{\theta_1 + \theta_2 = 90^\circ}$$

TRIANGLE

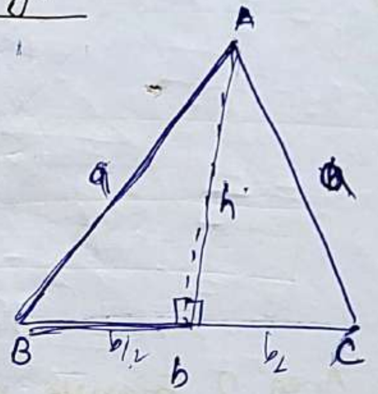
Scalene :- विषमबाहु त्रिभुज



$$S = \frac{a+b+c}{2}$$

$$\boxed{A = \sqrt{S(S-a)(S-b)(S-c)}}$$

Isoscles Triangle :-



$$h = \sqrt{a^2 - \left(\frac{b}{2}\right)^2}$$

$$= \frac{\sqrt{4a^2 - b^2}}{2}$$

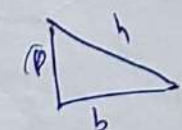
$$A = \frac{1}{2} \times \frac{\sqrt{4a^2 - b^2}}{2} \times b$$

$$\boxed{A = \frac{b}{4}\sqrt{4a^2 - b^2}}$$

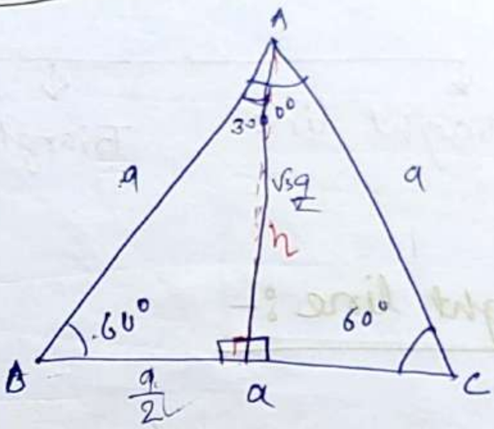
$h^2 = b^2 + p^2$

$p^2 = h^2 - b^2$

$p = \sqrt{h^2 - b^2}$



Equilateral Triangle :-



$P = 3a$

$\frac{1}{2} \times a \times \frac{\sqrt{3}a}{2}$

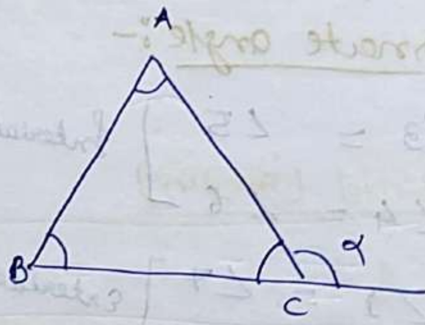
$$\text{Area} = \frac{\sqrt{3}}{4} a^2$$

$$\boxed{\frac{\sqrt{3}}{4} a^2}$$

$$\boxed{h = \frac{\sqrt{3}}{2} a}$$

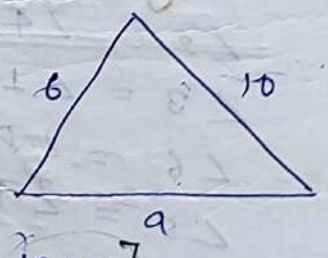
Property of Triangle :-

Rule 1 :-



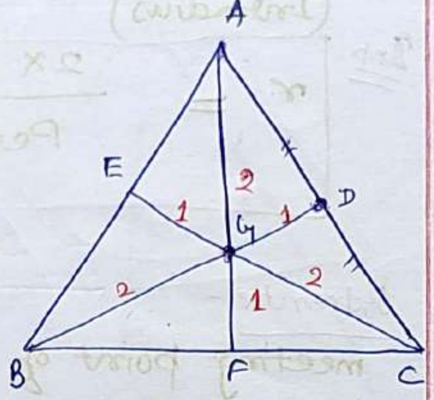
$$\boxed{\alpha = A + B}$$

Rule 2 :-



$$[10 - 6 < a < 10 + 6]$$

Medians :- (माध्यिका)



→ Median is a line from vertex of one triangle to midpoint of opposite side.

→ Median devides triangle in two equal part.

$\boxed{Ar.\ of\ ABF = Ar.\ of\ AFC}$

$= \frac{1}{2}\ Ar.\ of\ ABC$

centroid (केन्द्रक)

→ Meeting point of medians are centroid.

→ centroid divides each medians in 2:1 ratio.

$AG : GF = 2:1$

$CG : GE = 2:1$

$BG : GD = 2:1$

→ All 6 small triangles have equal area to

$$\boxed{Ar.\ 1, 2, 3 .. = \frac{Area\ of\ \Delta ABC}{6}}$$

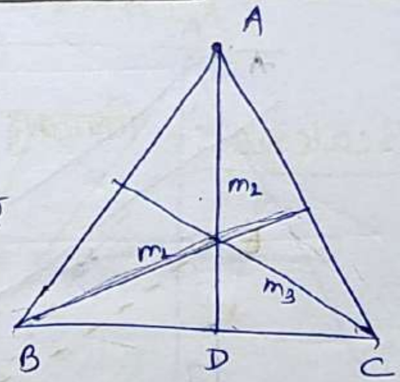
Imp ⇒ *

$$\boxed{3(AB^2+BC^2+AC^2) = 4(AD^2+BE^2+CF^2)}$$

$$\Rightarrow \frac{4}{3}(AD+BE+CF) > (AB+BC+AC)$$

Appllonius Theorem :-

Medians का length के लिए



$$AB^2 + AC^2 = 2(AD^2 + BD^2)$$

$$= 2(AD^2 + CD^2)$$

$$= 2\left(AD^2 + \frac{BC^2}{4}\right)$$

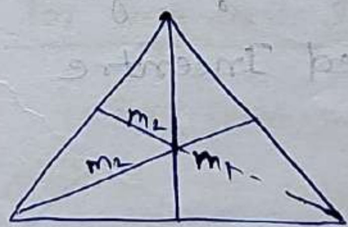
⇒ Let, m_1, m_2, m_3 are three medians of ΔABC then area of ΔABC =

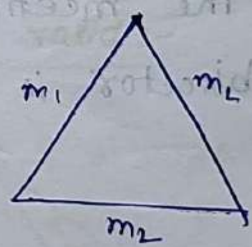
*

$$\boxed{Ar.\ of\ \Delta = \frac{4}{3}\sqrt{s(s-m_1)(s-m_2)(s-m_3)}}$$

$$s = \frac{m_1+m_2+m_3}{2}$$

⇒ ** Area of Triangle made by Medians of a triangle



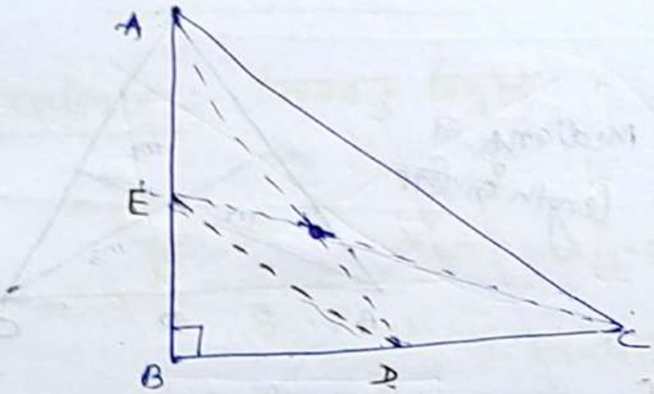


Area = A

⇓

$$\boxed{Area_{Med} = \frac{3}{4}\ Ar.\ of\ ABC}$$

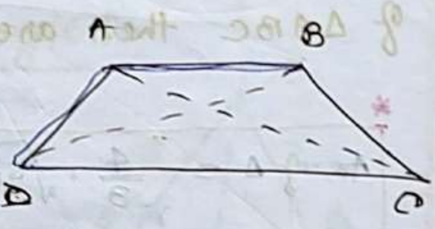
* Let the triangle is right angle Δ



AD & CE are Medians then,

$$\boxed{4(AD^2 + CE^2) = 5AC^2}$$

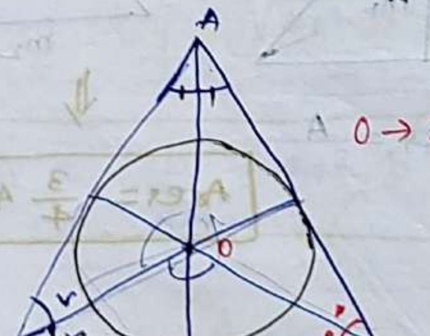
$$\boxed{ED^2 + AC^2 = AD^2 + CE^2}$$



$$\boxed{AB^2 + CD^2 = AC^2 + BD^2}$$

Incentre :-

The meeting point of angle bisector is called Incentre



O → Incentre

$$\angle BOC = 90 + \frac{\angle A}{2}$$

$$\angle COA = 90^\circ + \frac{\angle B}{2}$$

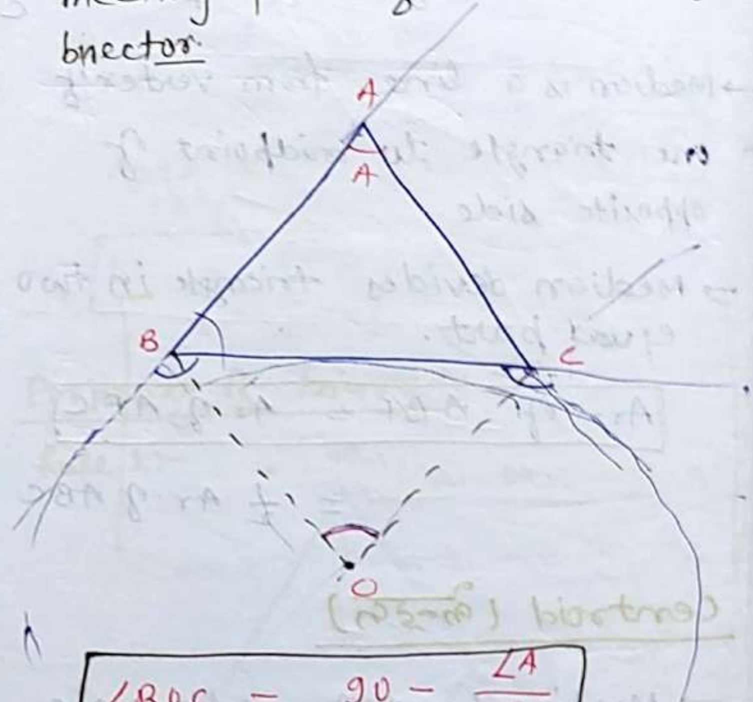
$$\angle AOB = 90^\circ + \frac{\angle C}{2}$$

Radius of Circle (Inradius)

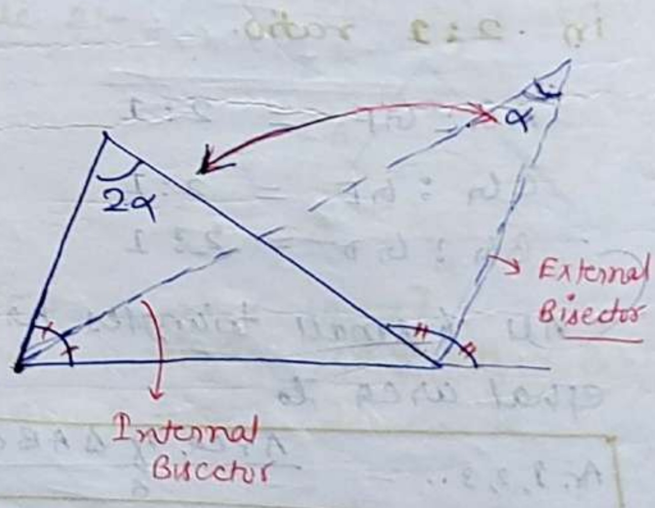
Imp
$$\boxed{r = \frac{2 \times \text{Area of } \Delta ABC}{\text{Perimeter of } ABC}}$$

Outcentre :-

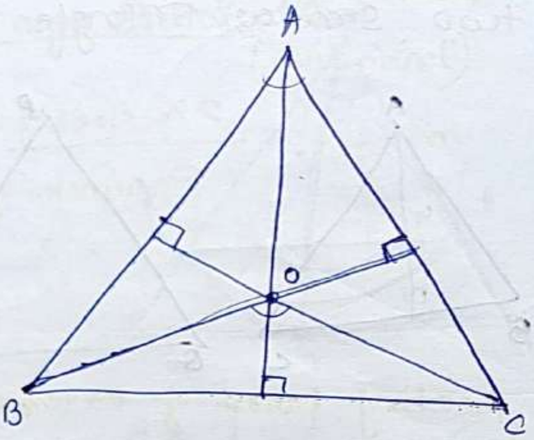
meeting point of External angle bisector



$$\boxed{\angle BOC = 90 - \frac{\angle A}{2}}$$



Orthocentre :-

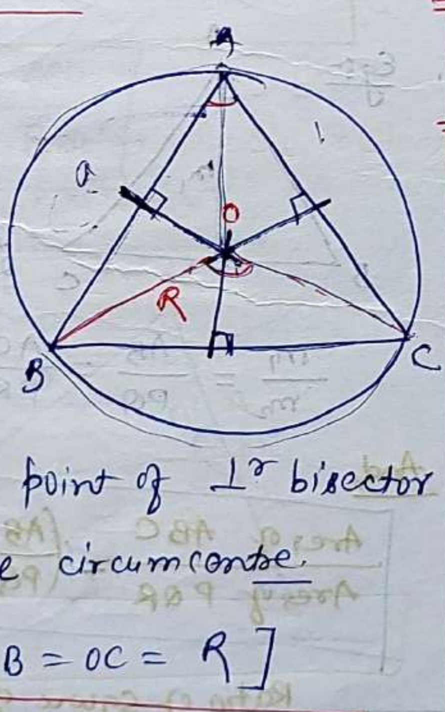


The meeting point of altitude is called orthocentre.

If 'O' is orthocentre then,

$$\left[\begin{array}{l} \angle BOC + \angle BAC = 180^\circ \\ \angle AOC + \angle ABC = 180^\circ \\ \angle AOB + \angle ACB = 180^\circ \end{array}\right]$$

Circumcentre :-



The meeting point of $\perp^r$ bisector of sides are circumcentre.

$$[OA = OB = OC = R]$$

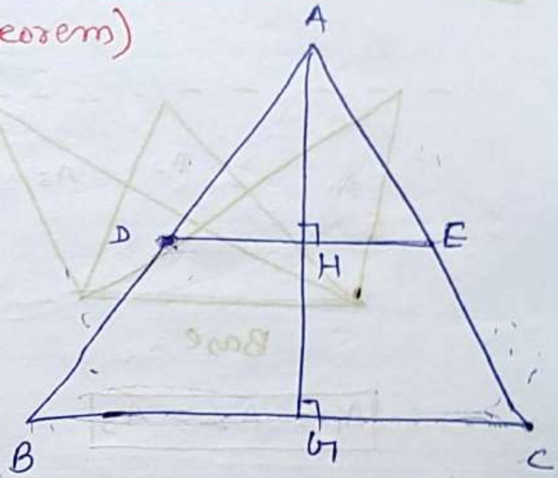
$$\boxed{R = \frac{a \times b \times c}{4 \times Ar.\ of\ \Delta ABC}}$$

$$\angle BOC = 2\angle A$$

$$\angle AOC = 2\angle B$$

Basic Propotionality Theorem:-

(BPT Theorem)



Let ABC be any triangle DE ∥ BC.

$$\Rightarrow \frac{AD}{AB} = \frac{AH}{AG} = \frac{AE}{AC} = \frac{DE}{BC}$$

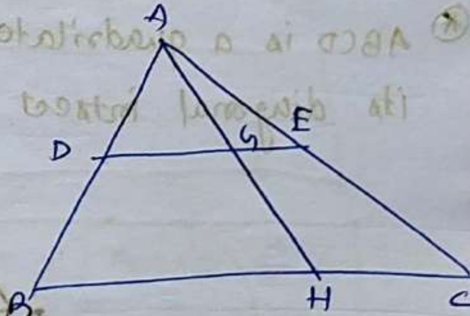
$$\Rightarrow \frac{AD}{DB} = \frac{AH}{HG} = \frac{AE}{EC}$$

$$\Rightarrow \frac{AD}{DE} = \frac{AB}{BC}$$

$$\Rightarrow \frac{AE}{ED} = \frac{AC}{CB}$$

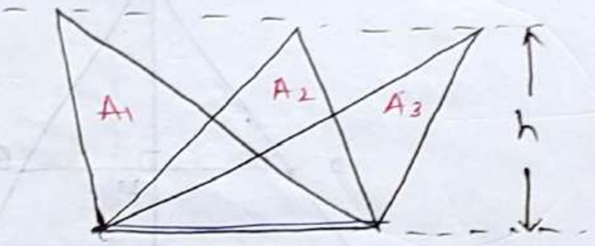
$$\Rightarrow \frac{AD}{AB} = \frac{DH}{BG}$$

For any line It is valid :-



$$\frac{AD}{AB} = \frac{AG}{AH} = \frac{AE}{AC} = \frac{DE}{BC} = \frac{DG}{BH}$$

Note:-

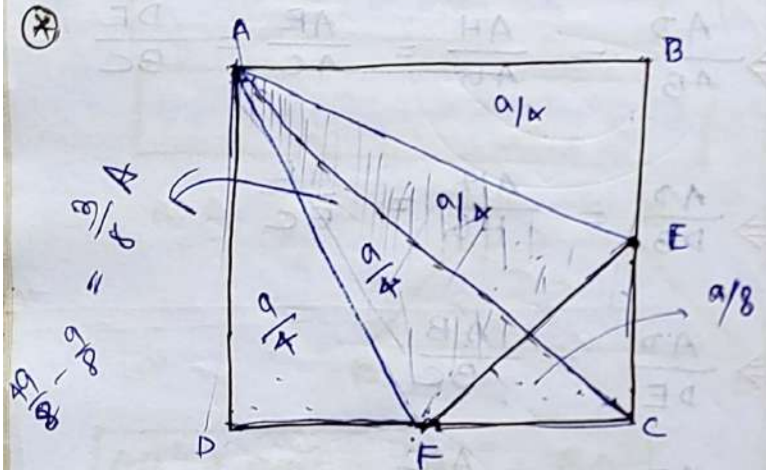


Base

$\boxed{A_1 = A_2 = A_3}$

:- with common base if height is same then Area is same.

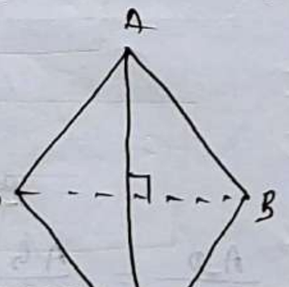
(*)



[Ar of AEF = $\frac{3}{8}$ × Area of ABCD]

[Ar. of CEF = $\frac{1}{8}$ × Area of ABCD]

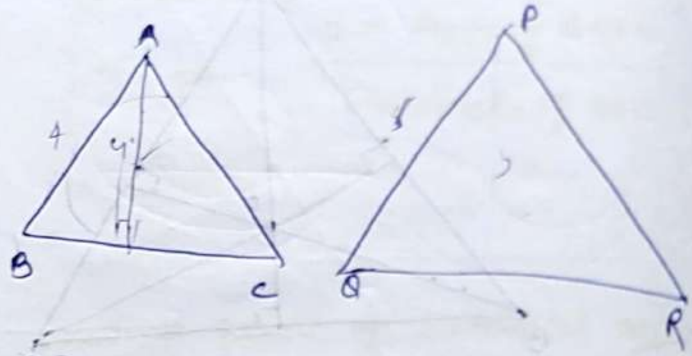
(*) ABCD is a quadrilateral and if its diagonal intersect at 90° then,



$\boxed{AB^2 + CD^2 = AD^2 + BC^2}$

V.V.I

(*) Let ABC and PQR are two similar triangle then:-



$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR} =$

= Ratio of their medians

= Ratio of their Altitude

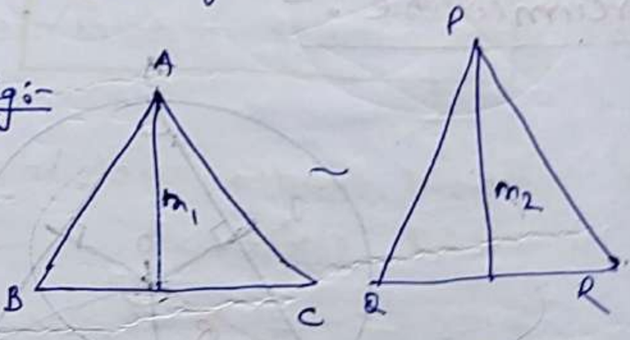
= Ratio of their angle bisector

= Ratio of their perimeter

= Ratio of their Inradii

= Ratio of their circum radii

Eg:-



$\frac{m_1}{m_2} = \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$

And,

$$\boxed{\frac{\text{Area of } ABC}{\text{Area of } PQR} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2}$$

= Ratio of square of Medians

= Ratio of square of Angle bisector

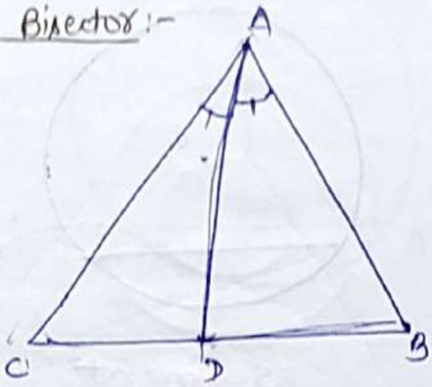
= Ratio of square of perimeter

= Ratio of square of Inradii

= Ratio of square of circum radii

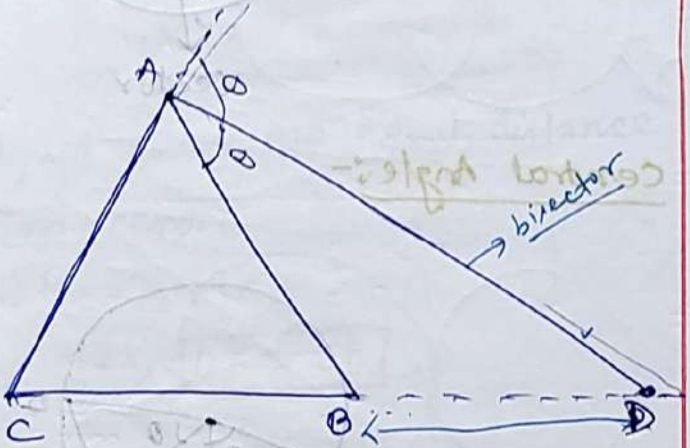
⊛ Angle Bisector Theorem:-

Internal angle Bisector:-



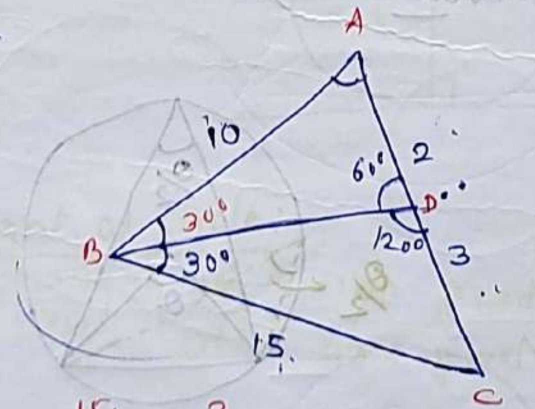
$$\boxed{\frac{AC}{AB}=\frac{CD}{DB}}$$

External Angle Bisector:-



$$\boxed{\frac{AC}{AB}=\frac{CD}{BD}}$$

Eg:-



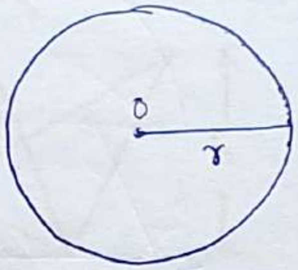
$$\frac{15}{10}=\frac{3}{2}$$

BD is angle bisector of $\angle B$

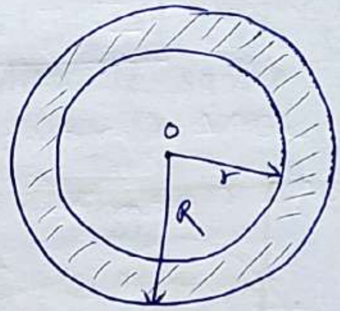
CIRCLE

$A = \pi r^2$

$C = 2\pi r$

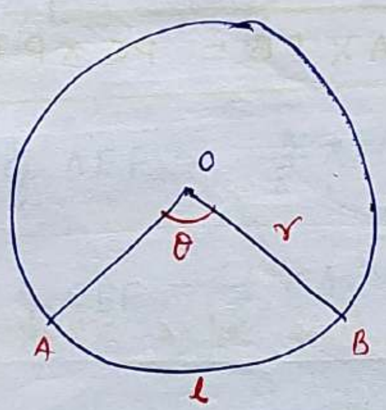


Concentric circle :-



Area of shaded part

$$[A = \pi (R^2 - r^2)]$$



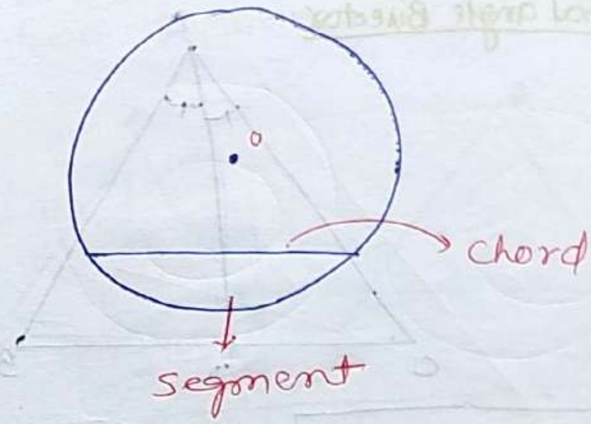
$$\boxed{\theta = \frac{l}{r}}$$

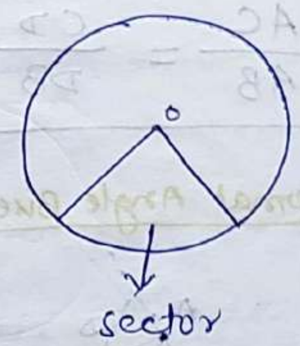
$$\boxed{l = \frac{2\pi r \times \theta}{360}}$$

Area of OlAB $= \dfrac{\pi r^2}{360} \times \theta$

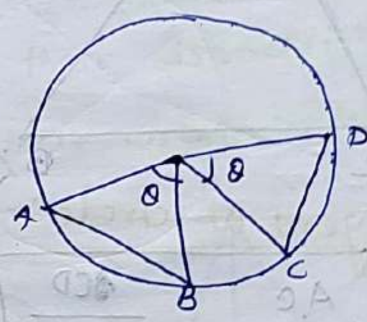
or, $A = \dfrac{r \times l}{2}$

$P = (l + r + r)$



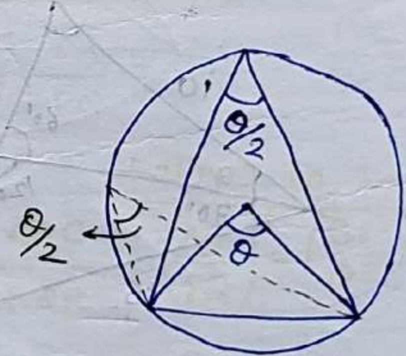


Central Angle :-



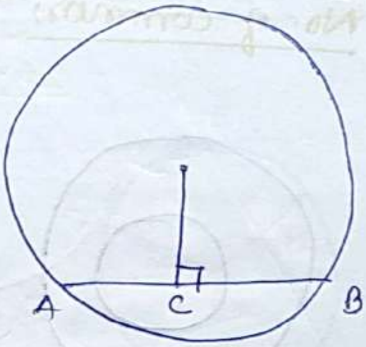
$\overset{\frown}{AB} = \overset{\frown}{CD}$

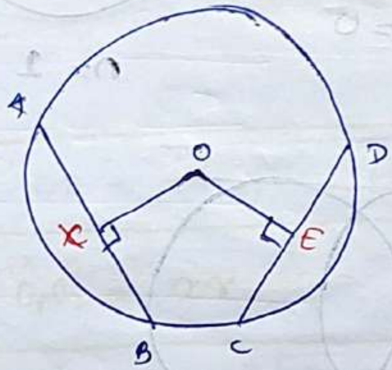
Equal chord makes equal angle at centre.



Angle made by a chord at centre is twice the angle made at major segment.

$AC = CB$



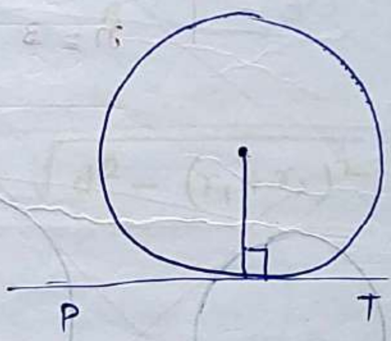


→ Equal chord are equal distance from centre.

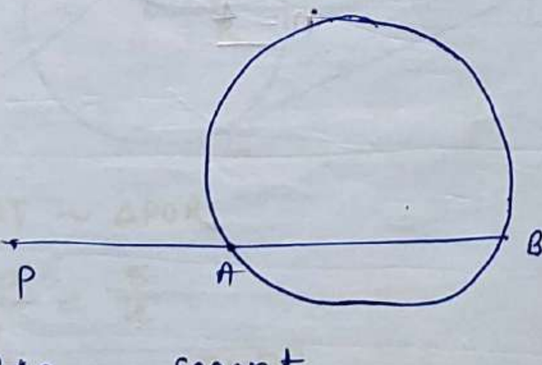
If $AB = CD$

then, $\boxed{OX = OE}$

Tangent at circle :-

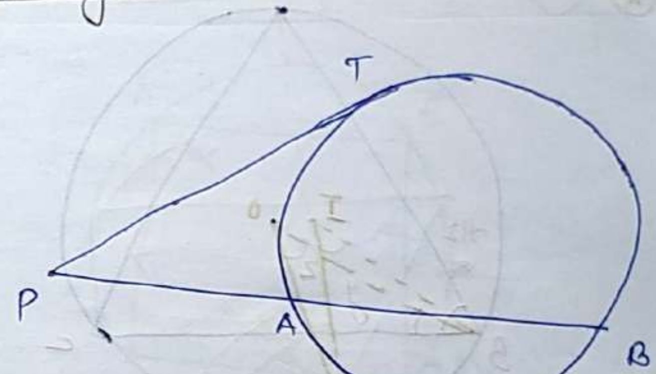


Secant at circle :-

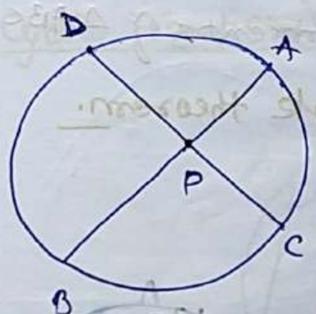


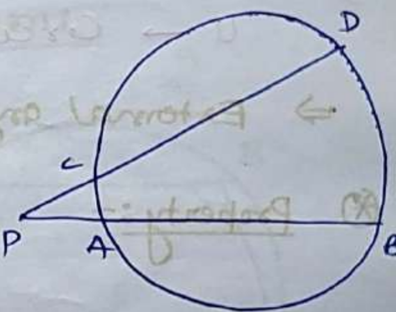
PAB = secant

Property :-



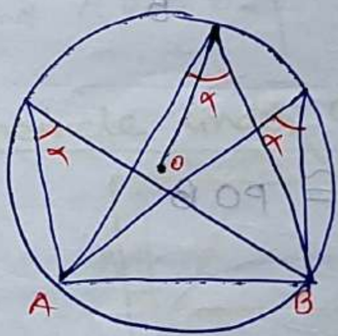
$$\boxed{PT^2 = PA \times PB}$$





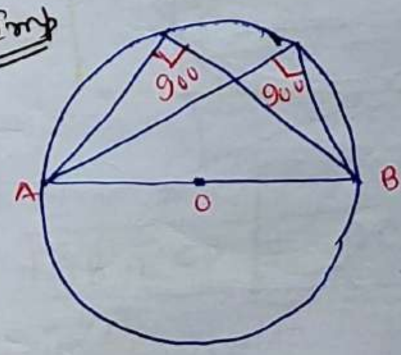
$$\boxed{PA \times PB = PC \times PD}$$

Property :-



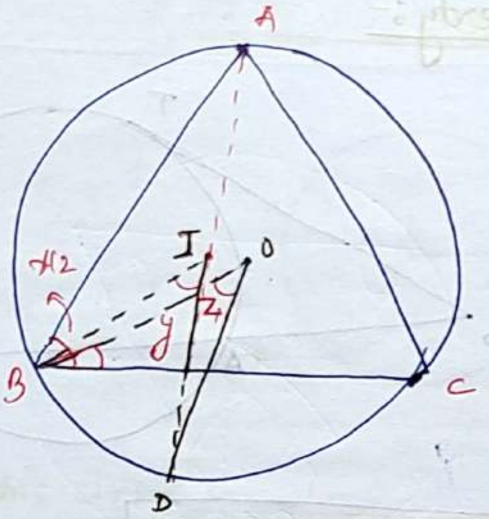
→ Angle made by same chord are same at major segment

Imp



Angle made by chord at either segment is 90°.

⊛ (F*)



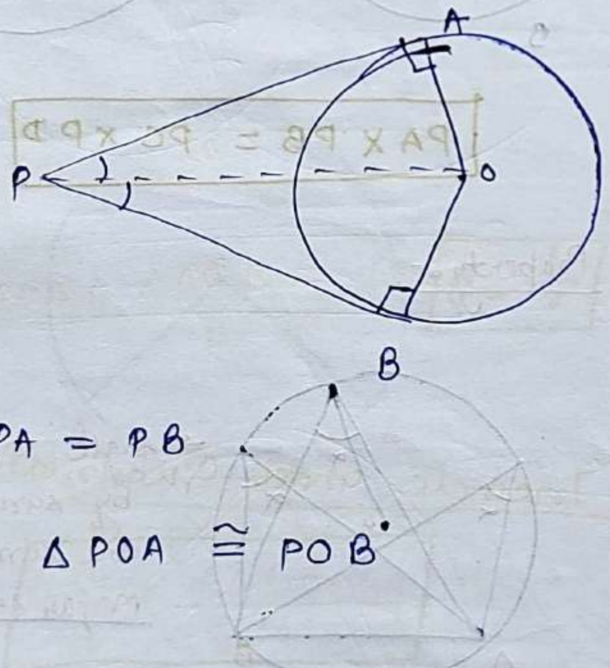
$$\boxed{y = \frac{x}{2} + \frac{z}{2}}$$

I → Incentre of ΔABC

O → Circumcentre of ΔABC

⇒ External angle theorem.

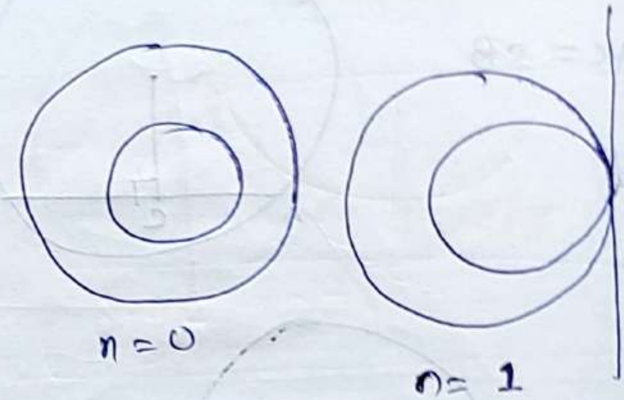
⊛ Property :-



$PA = PB$

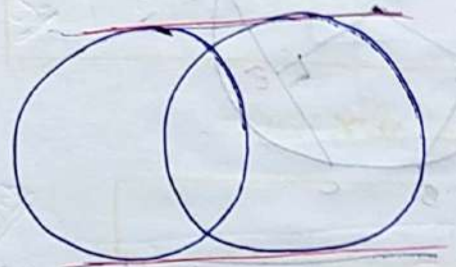
And, $\Delta POA \cong POB$

No. of common Tangent :-

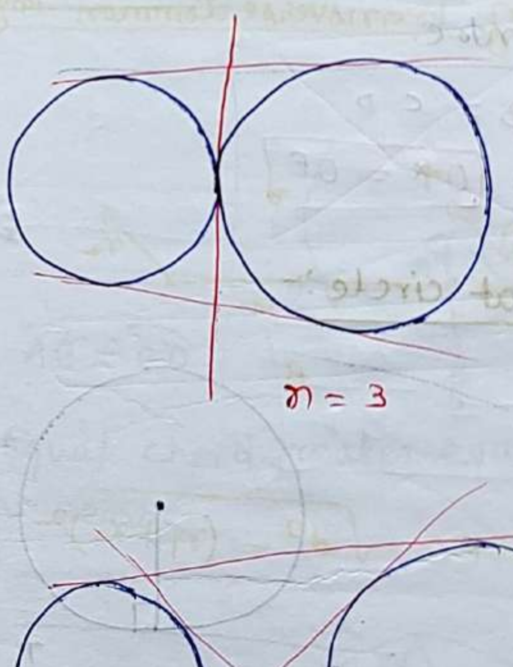


$n = 0$

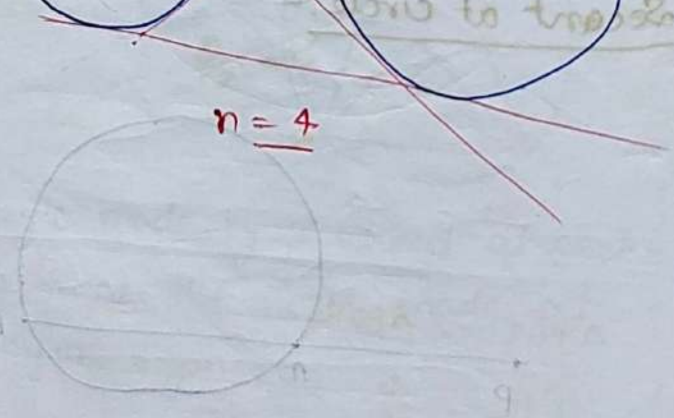
$n = 1$



$n = 2$

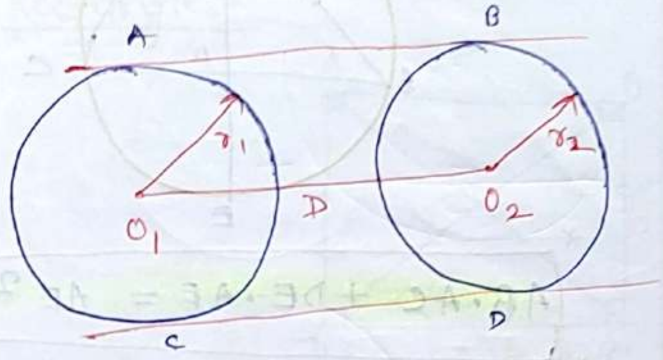


$n = 3$



$n = 4$

* Length of Direct common Tangent

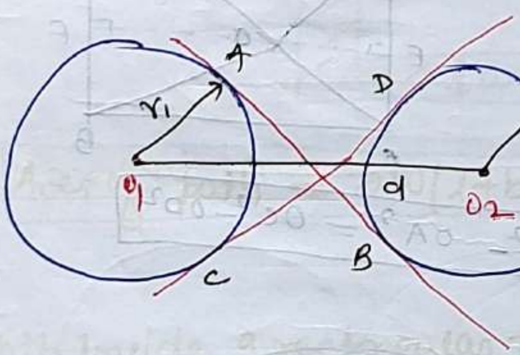


$$\boxed{AB = CD = \sqrt{d^2 - (r_1 - r_2)^2}}$$

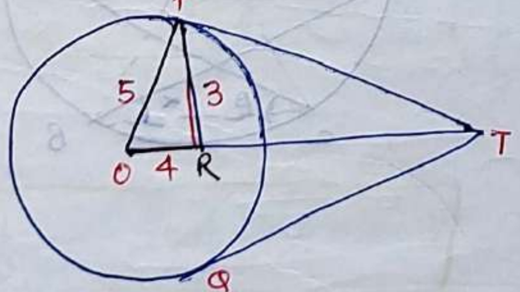
If $O_1O_2 = 2r$

$$\boxed{AB = CD = 2\sqrt{r_1 r_2}}$$

⇒ Length of Transverse Common Tangent



$$\boxed{AB = CD = \sqrt{d^2 - (r_1 + r_2)^2}}$$

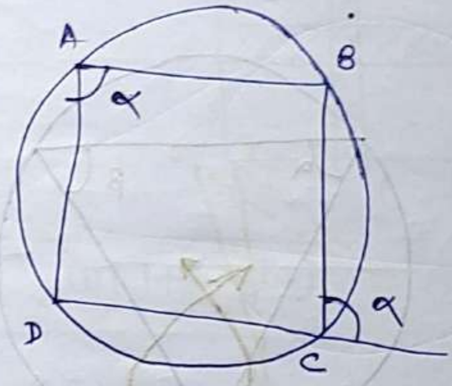


$\Delta POT \sim \Delta POR$

$$\frac{TP}{3} = \frac{5}{4}$$

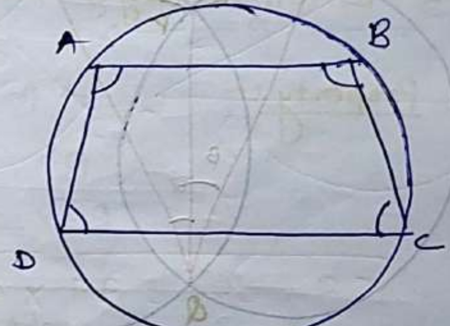
$$\boxed{TP = \frac{5 \times 3}{4}}$$

* Cyclic Quad :-



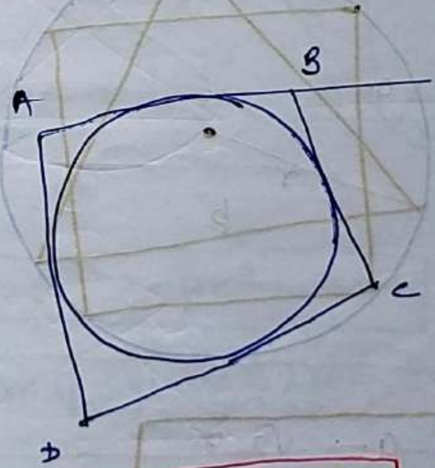
$$\boxed{\begin{aligned} \angle A + \angle C &= 180^\circ \\ \angle B + \angle D &= 180^\circ \end{aligned}}$$

Cyclic Trapezium :-



$\left.\begin{array}{l} \angle A = \angle B \\ \angle D = \angle C \\ \text{And } AD = BC \end{array}\right\}$

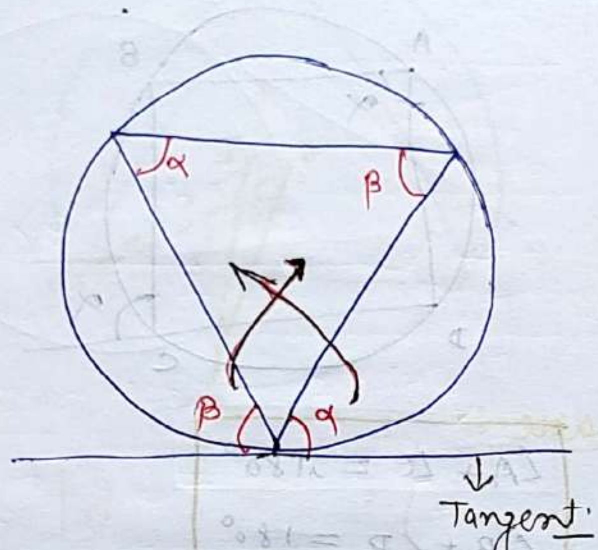
Incircle under Quadrilateral :-



$$\boxed{AB + CD = BC + AD}$$

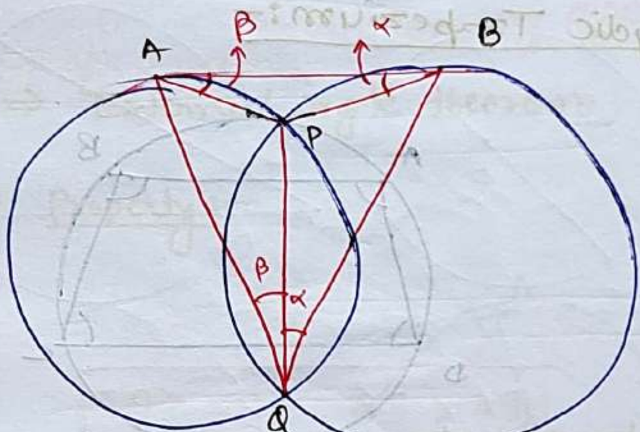
Sum of opposite sides are equal.

Alternate Segment Theorem:-



⇓

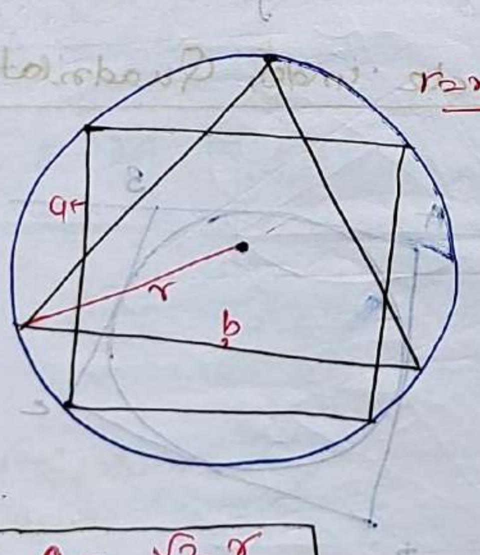
Q.



$$\boxed{\angle APB + \angle AQB = 180^\circ}$$

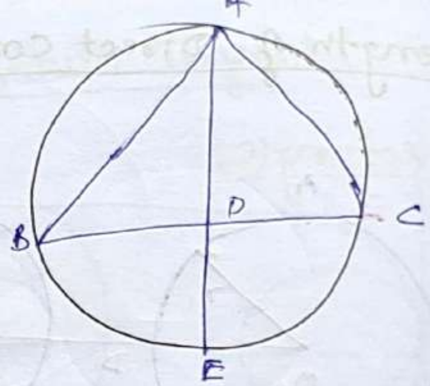
(*)

r = radius



□	$a = \sqrt{2}\, r$
△	$b = \sqrt{3}\, r$

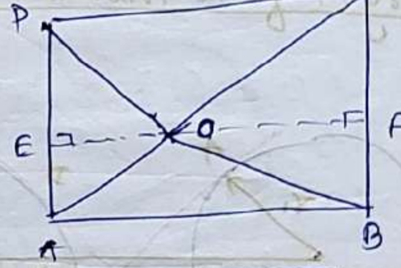
(*)



$$\boxed{AB \cdot AC + DE \cdot AE = AE^2}$$

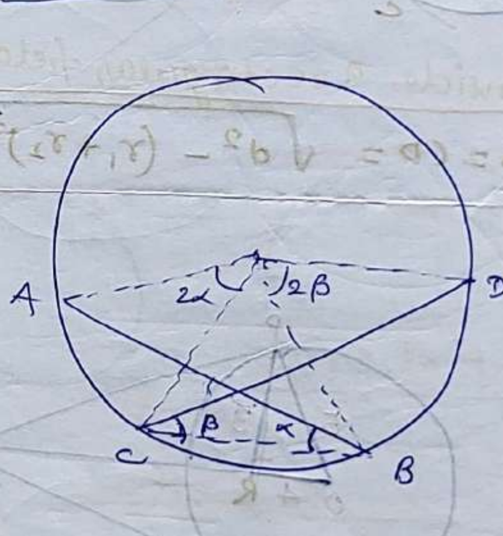
Q. A point O is interior of a rectangle ABCD is joined with each other of the vertices A, B, C and D. then :-

Imp (a) $\boxed{OB^2 - OA^2 = OC^2 - OD^2}$



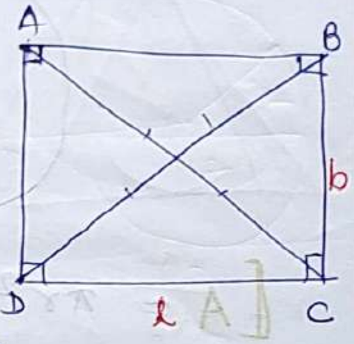
$$\boxed{OA^2 - OA^2 = OC^2 - OD^2}$$

Q



Mensuration

Rectangle:-

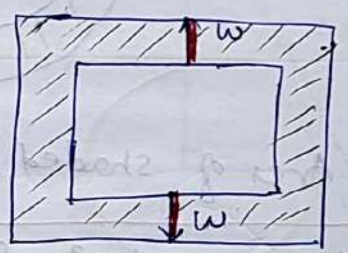


Area = $l \times b$

Perim = $2(l+b)$

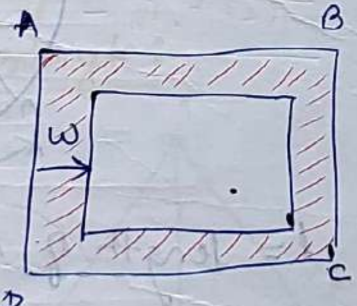
$AB = BD = \sqrt{l^2+b^2}$

Path outside a rectangular field:-

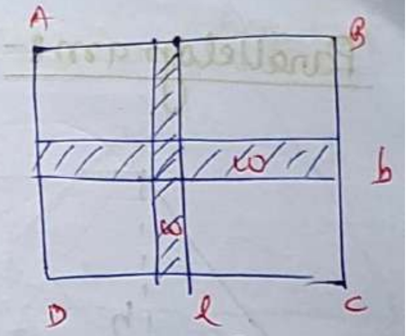


$\boxed{\text{Area of path} = 2w(l+b+2w)}$

Path Inside a rectangular field:-

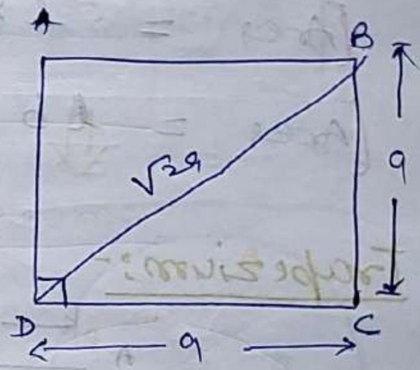


$\boxed{\text{Area of Path} = 2w(l+b-2w)}$



$\boxed{A = w(l+b-w)}$

Square:-

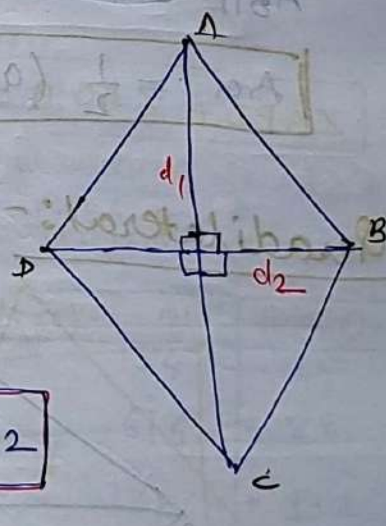


Area = a^2

Perimeter = $4a$

Diagonal = $\sqrt{2}\,a$

Rhombus:-



$\boxed{\text{Area} = \frac{1}{2} d_1 d_2}$

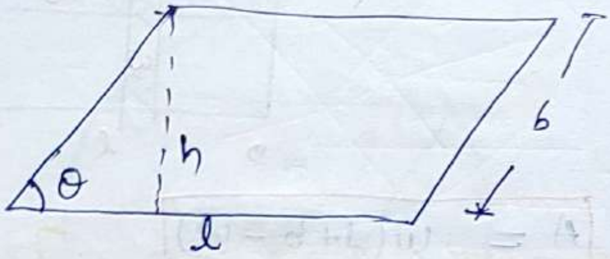
$\boxed{AB^2 + DC^2 = BC^2 + AD^2}$

⇒ sum of square of side = sum of sq. of Diagonal

★ $\Delta AOB = \Delta BOC = \Delta COD = \Delta DOA$

each equal $\frac{1}{4}$th of area of rhombus

Parallelogram :-

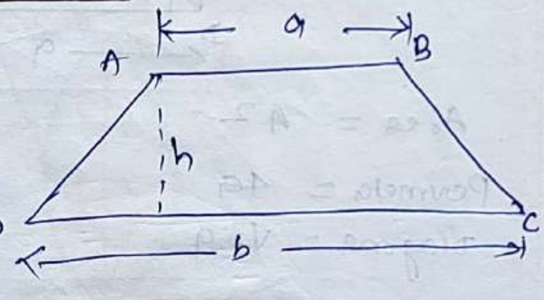


[Perimeter = $2(l+b)$]

[Area = $l \times h$]

[Area = $lb \sin\theta$]

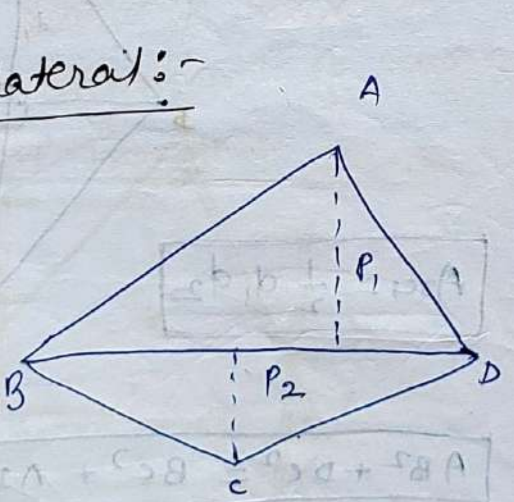
Trapezium :-



AB || CD

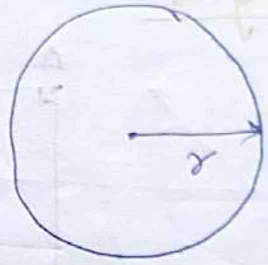
$\boxed{\text{Area} = \frac{1}{2}(a+b) \times h}$

Quadilateral :-



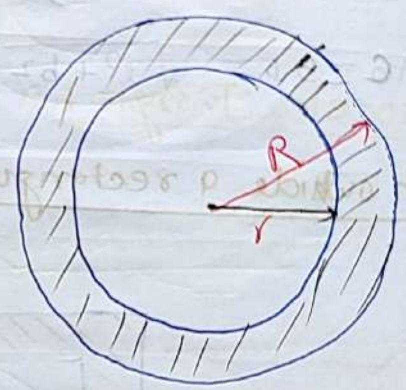
$\boxed{\text{Area of } ABCD = \frac{1}{2} BD (P_1 + P_2)}$

Circle :-



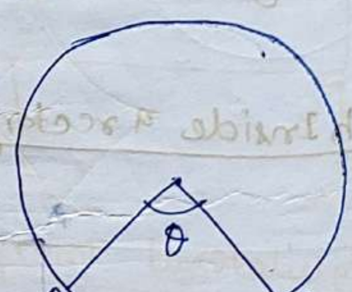
[$A = \pi r^2$]

[$C = 2\pi r$]



Area of shaded part

$\boxed{A = \pi(R^2 - r^2)}$



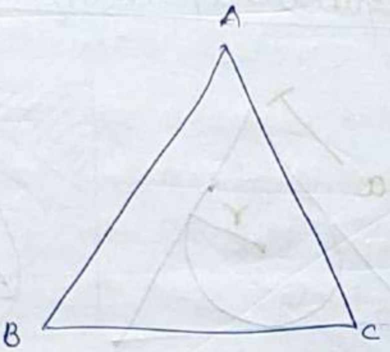
l = length of AB

$\boxed{l = \frac{2\pi r}{360} \times \theta}$

$\boxed{A_1 = \frac{\pi r^2}{360} \times \theta}$

[$A_1 = \frac{lr}{2}$]

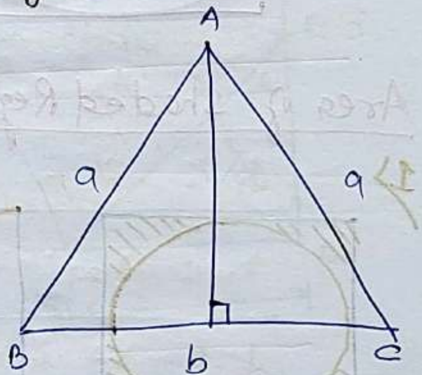
Triangle :-



$$s = \frac{a+b+c}{2}$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Isoceles Triangle :-

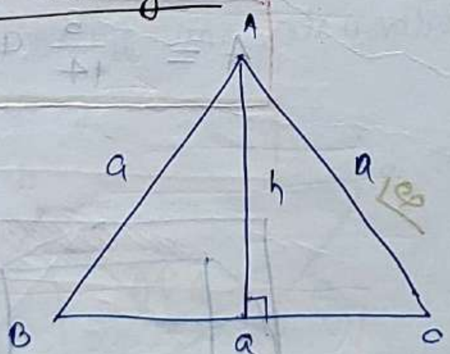


$$\boxed{P = 2a + b}$$

$$\boxed{A = \frac{b}{4}\sqrt{4a^2 - b^2}}$$

$$\left[h = \frac{\sqrt{4a^2 - b^2}}{2}\right]$$

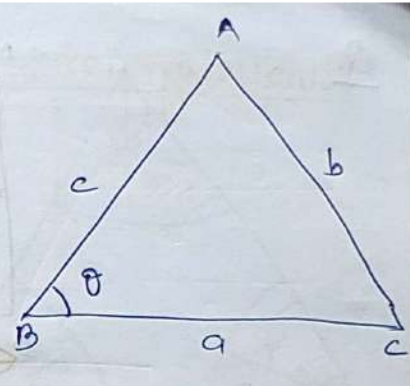
Eq uilateral Triangle :-



$$\boxed{h = \frac{\sqrt{3}}{2}a}$$

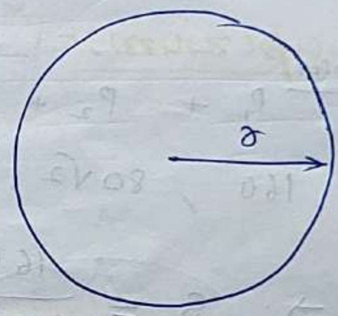
$$\boxed{A = \frac{\sqrt{3}}{4}a^2}$$

Note :-



$$\left[\begin{array}{l} Ar.\ of\ \Delta ABC = \frac{1}{2}ab\sin C \\ Ar.\ of\ \Delta ABC = \frac{1}{2}bc\sin A \\ Ar.\ of\ ” = \frac{1}{2}ca\sin B \end{array}\right]$$

Remember :-



When $r = 7\,cm$

$\Rightarrow A = \boxed{154 \text{ unit}^2}$

$r = 14\,cm$

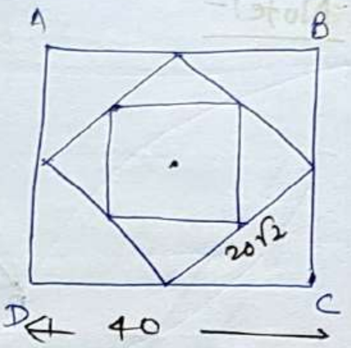
$\Rightarrow A = 616 \text{ unit}^2$

$r = 21\,cm$

$A = 1386 \text{ unit}^2$

r	(Area) A	Perim (P)
7 →	154 →	44
14 →	616 →	88
21 →	1386 →	132

Q.



Find Sum of Perimeter of Squres:-

G.P

$a, ar, ar^2, ar^3 \ldots ar^{(n-1)}$

$$\boxed{S_\infty = \frac{a}{1-r}} \quad \left\{ r = \frac{T_2}{T_1} \right\}$$

There,

$P_1 + P_2 + P_3 \ldots$

$160, 80\sqrt{2}, 80, 40\sqrt{3} \ldots$

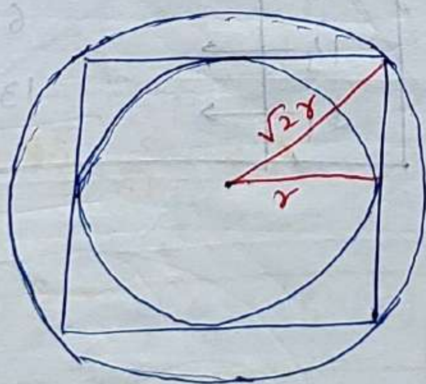
$$\Rightarrow S = \frac{160}{1-\frac{1}{\sqrt{2}}} = \frac{160\sqrt{2}}{\sqrt{2}-1}$$

$$\boxed{S = 160(\sqrt{2}+2)}$$

Note:-

Ratio of radius of Incire to that of circumcircle of square, equilateral Triangle :-

Square:-



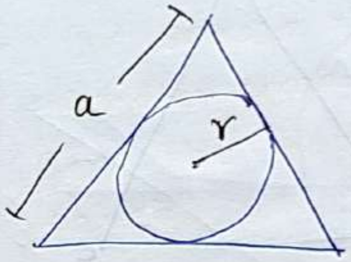
$r : \sqrt{2}r$

$\boxed{1 : \sqrt{2}}$

$A_1 : A_2$

$\boxed{1 : 2}$

Equilateral Triangle:-



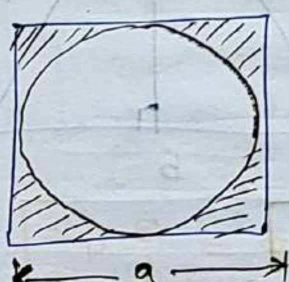
$r = \frac{a}{2\sqrt{3}}$: $R = \frac{a}{\sqrt{3}}$

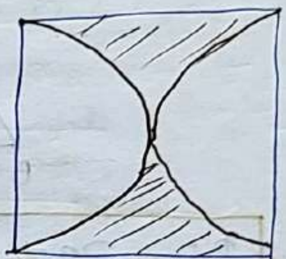
$$\boxed{r : R = 1 : 2}$$

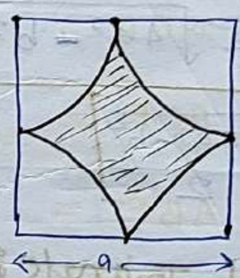
$$\boxed{A_1 : A_2 = 1 : 4}$$

Area of Shaded Region :-

1)

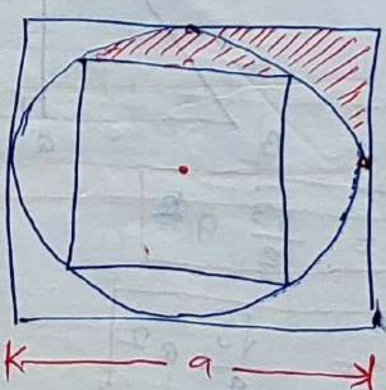




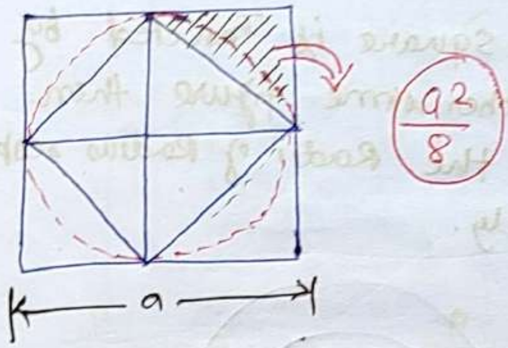


$$\boxed{A = \frac{3}{14} a^2}$$

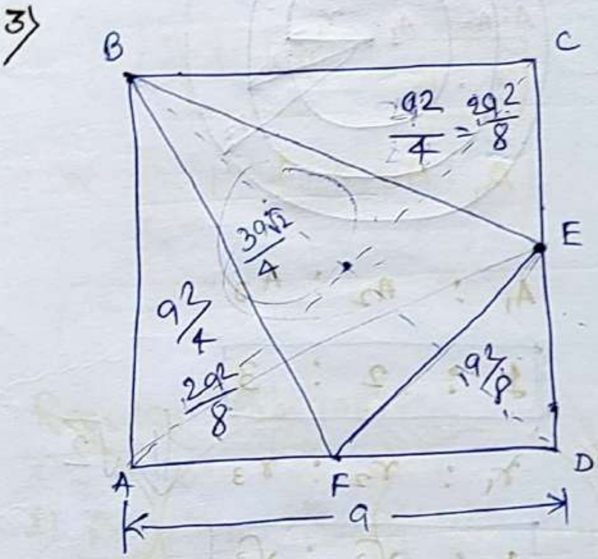
2)



Rotate the figure:-

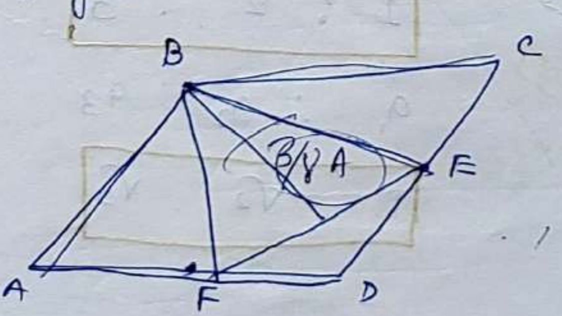


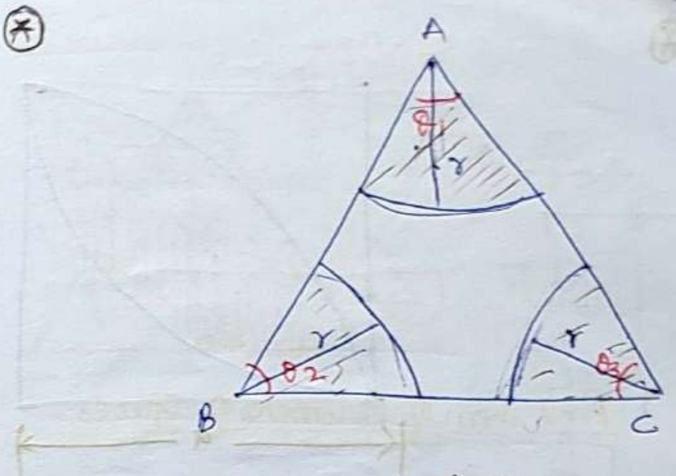
3)



$$\text{Area of } \Delta BEF = \frac{3}{8} a^2$$
$$\text{Area of } \Delta DEF = \frac{a^2}{8}$$
$$\text{Area of } \Delta BCE = \frac{a^2}{4}$$
$$\text{Area of } \Delta ABF = \frac{a^2}{4}$$

All these formulas are also valid in ||gm.





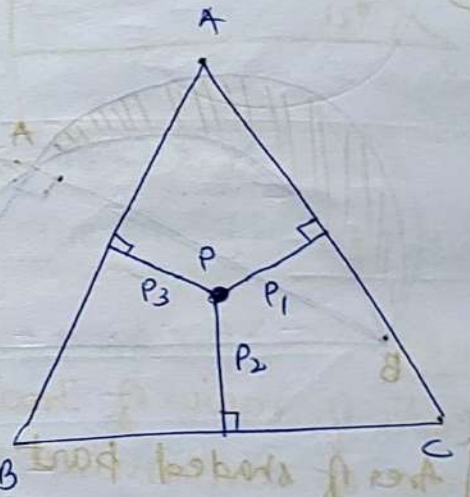
Area of shaded Area.

$$A = \frac{\pi r^2}{360} * (\theta_1 + \theta_2 + \theta_3)$$

$$A = \frac{\pi r^2}{360} (180^\circ)$$

$$\boxed{A = \frac{\pi r^2}{2}}$$

$\frac{\sqrt{3}}{2} a = h$



From a point ⊥r are drawn to each sides then Area of Triangle in terms of P_1 P_2 P_3.

$$\boxed{a = \frac{2}{\sqrt{3}} (P_1 + P_2 + P_3)}$$

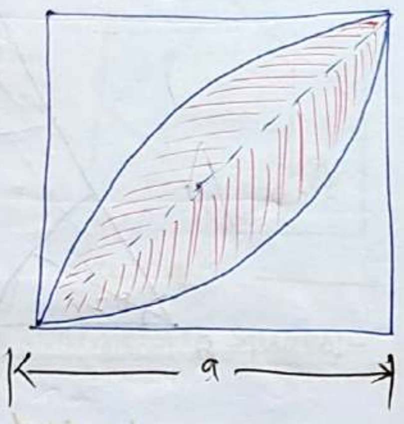
side

$$\because \frac{\sqrt{3}}{2} a = (P_1 + P_2 + P_3)$$

$$\boxed{a = \frac{2}{\sqrt{3}} (P_1 + P_2 + P_3)}$$

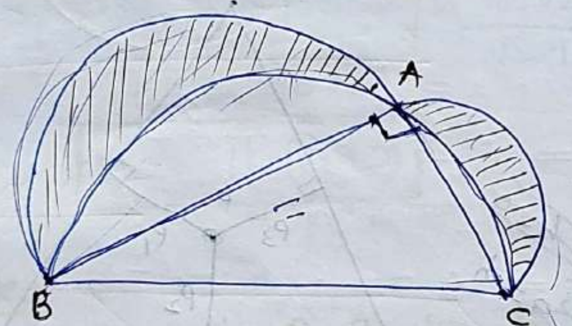
(Area $= \frac{\sqrt{3}}{4} a^2$

(*)



Shaded	Unshaded
4 :	3
$\frac{4}{7}a^2$	$\frac{3}{7}a^2$

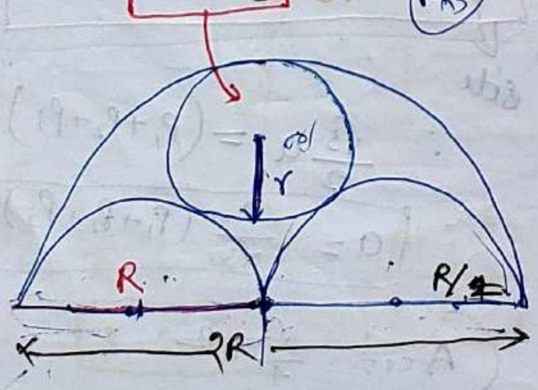
(*)



[Area of shaded part = Area of ΔABC]

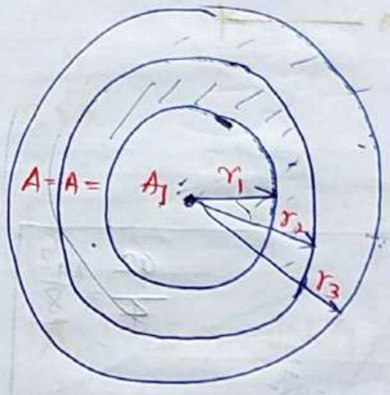
(*)

$\boxed{r = \frac{R}{3}}$



<u>Note:-</u>

A circle, equilateral triangle and square is Trisected by another same figure then find the Radii of Radius respectively.

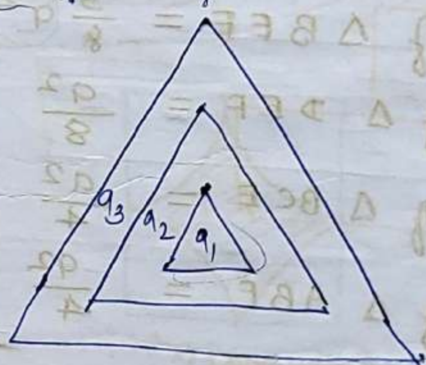


$A_1 : A_2 : A_3$

$\boxed{1 : 2 : 3}$ $\Downarrow \sqrt{2}$

$r_1 : r_2 : r_3$

$\boxed{1 : \sqrt{2} : \sqrt{3}}$

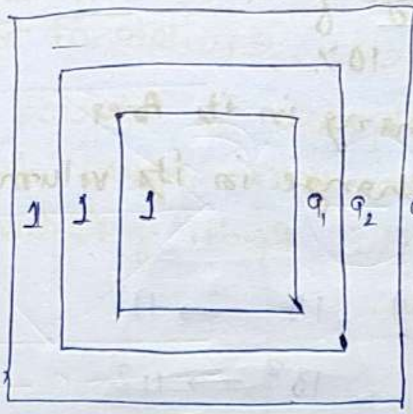


$A_1 : A_2 : A_3$

$\boxed{1 : 2 : 3}$

$a_1 : a_2 : a_3$

$\boxed{1 \quad \sqrt{2} \quad \sqrt{3}}$



$\Rightarrow A_1 : A_2 : A_3$

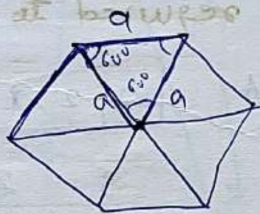
$\boxed{1 : 2 : 3}$

$\Rightarrow a_1 : a_2 : a_3$

$\boxed{1 : \sqrt{2} : \sqrt{3}}$

Regular Polygon:-

1) Regular Hexagon:-



$\boxed{P = 6a}$

$\boxed{A = \frac{\sqrt{3}}{4} a^2 \times 6}$

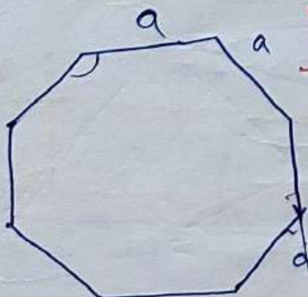
Sum of

Regular Octagon:-

$\boxed{\text{Sum of Interior Angle} = 180(n-2)}$

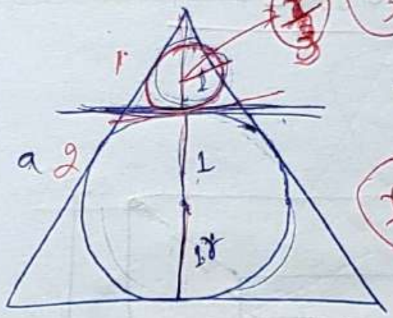
$\boxed{180(n-2)}$

$= (2n-4) 90^\circ$



$\boxed{P = 8a}$

$\boxed{\text{Area} = 2a^2(1+\sqrt{2})}$



r = inradius

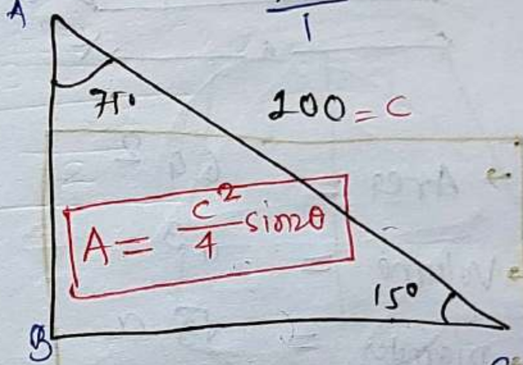
$1 : 3$

$\boxed{1 : 3}$

$\frac{r}{3}$

$\frac{2A}{s}$

$$\boxed{r = \frac{2 \times \text{Area of Triangle}}{\text{Perimeter}}}$$



$\frac{c^2}{1}$

$100 = c$

$$\boxed{A = \frac{c^2}{4} \sin 2\theta}$$

$AB = 100 \cos 75^\circ$

$BC = 100 \sin 75^\circ$

$\text{Area} = \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} 100 \sin 15 \times 100 \cos 15^\circ$

$= \frac{1}{2 \times 2} \times 100 \times 100 \,(2 \sin 15 \cos 15^\circ)$

$= \frac{1}{4} \times 100 \times 100 \times \sin 30^\circ$

$= \frac{1}{8} \times 1000 \times 100 = 1250$ Ans

Note:-

$\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$

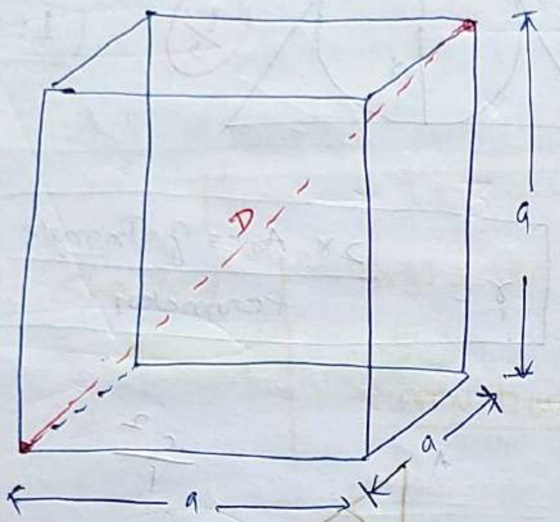
$\sin 75 = \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\tan 15 = 2 - \sqrt{3}$

$\tan 75 = 2 + \sqrt{3}$

Volume :-

Cube :-

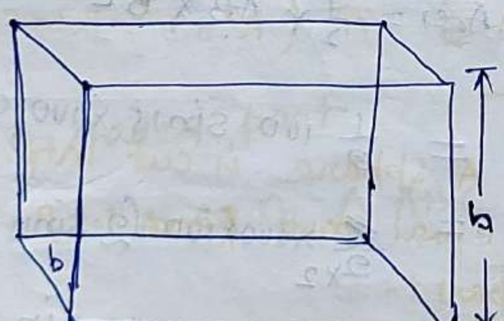


$\rightarrow$ Area $= 6a^2$

$\rightarrow$ Volume $= a^3$

$\rightarrow$ Diameter $= \sqrt{3}\ a$

Cuboid :-



$\left[\text{Area} = 2(lb + bh + hl)\right]$

$\left[\text{Volume} = lbh\right]$

$\left[D = \sqrt{l^2 + b^2 + h^2}\right]$

(*) Edge of a cube is increased by 10%.

(i) change in its Area

(ii) change in its volume.

A (i) $10 \longrightarrow 11$

$10^2 \longrightarrow 11^2$

$100 \longrightarrow 121$

21%

(ii) $10 \longrightarrow 11$

$10^3 \longrightarrow 11^3$

$100.0 \longrightarrow 133.1$

33.1% A

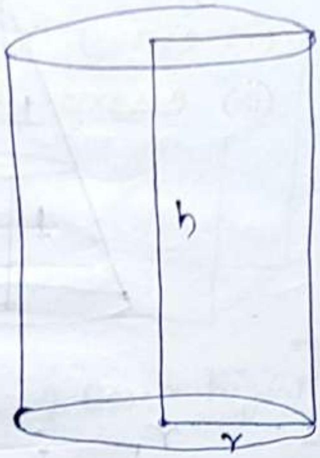
(**) (*) The dimension are 3, 2, 1, How many such block are required to make it a cube?

A $3, 2, 1 \Rightarrow 6$

$$\begin{array}{ccc} 3 & 2 & 1 \\ \times 2 & \times 3 & \times 6 \\ \| & \| & \| \\ 6 & 6 & 6 \end{array} \longrightarrow 36 \text{ Blocks}$$

A

Cylinder :-

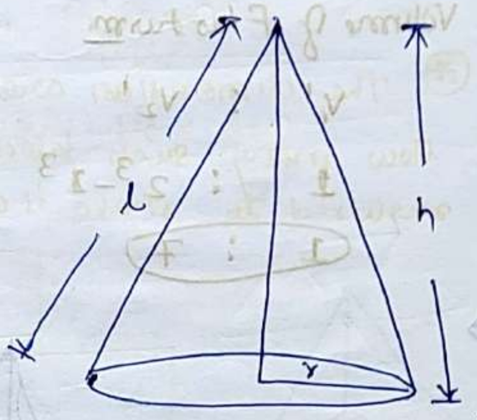


$$\boxed{V = \pi r^2 h}$$

$$\boxed{C.S.A = 2\pi r h}$$

$$\boxed{T.S.A = 2\pi r h + 2\pi r^2}$$

Cone :-

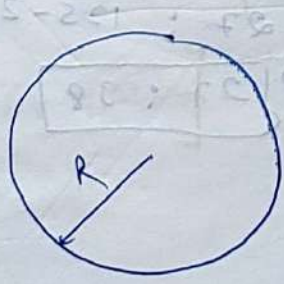


$$\boxed{V = \frac{1}{3}\pi r^2 h}$$

$$\boxed{C.S.A = \pi r l}$$

$$T.S.A = \pi r l + \pi r^2$$

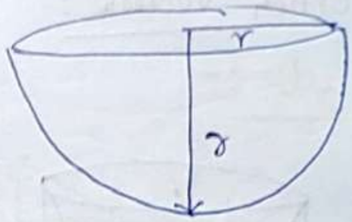
Sphare :-



$V = \frac{4}{3}\pi r^3$

$A = 4\pi r^2$

Semi-sphare :-



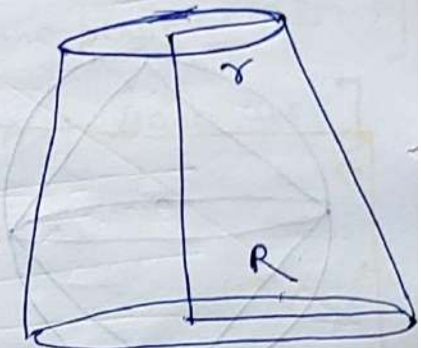
$V = \frac{2}{3}\pi r^2$

$C.S.A = 2\pi r^2$

$T.S.A = 3\pi r^2$

Flustrum :-

Imp

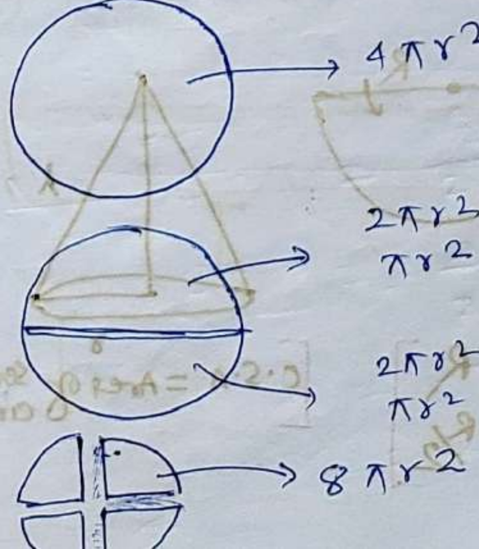


$$V = \frac{1}{3}\pi (R^2 + r^2 + rR) h$$

$$L.S.A = \pi (r + R) l$$

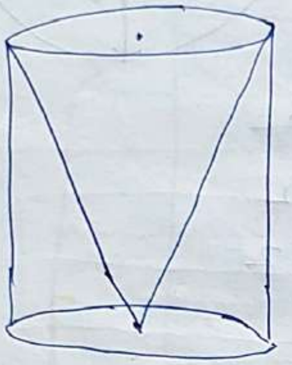
$$T.S.A = \pi (R^2 + r^2 + Rl + rl)$$

(*) A sphare is cut into four equal part find T.S.A of each part :-



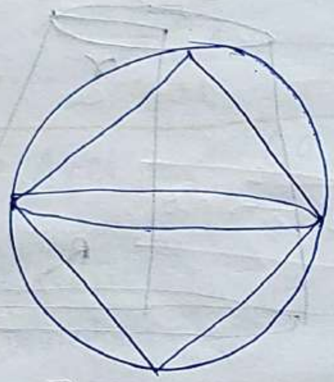
→ No of Cone can cut from other volume :-

Cone
↓



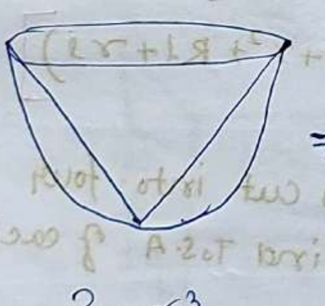
→ 3

$\pi r^2 h$ $\qquad$ $\frac{1}{3}\pi r^2 h$



⇒ 4

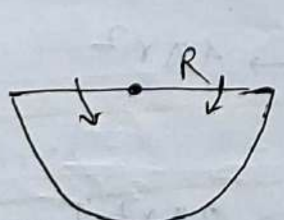
$\frac{4}{3}\pi r^3$ $\qquad$ $\frac{\pi r^2 \times r}{3}$

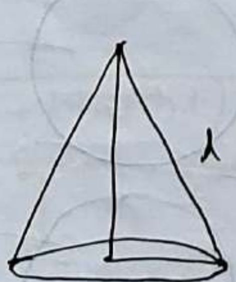


⇒ 2

$\frac{2}{3}\pi r^3$ $\qquad$ $\frac{1}{3}\pi r^3$

(*)

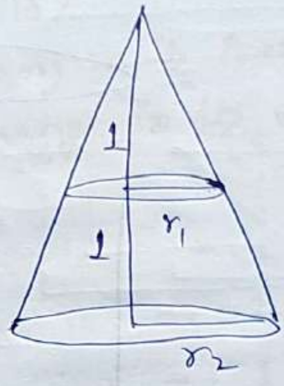




$$\begin{bmatrix} l = R \\ r = R/2 \end{bmatrix}$$

[C.S.A = Area of semi circle]

(*) Note:-



$r_1 : r_2$

$1 : \boxed{2}$

$\Rightarrow V_1 : \boxed{V_2}$

$\boxed{(r_1)^3 : (r_2)^3}$

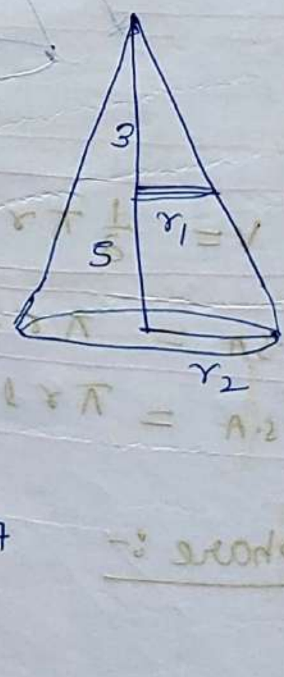
$\Rightarrow 1 : 8$

Volume of Flustum

$V_1' : V_2'$

$1 : 2^3 - 1^3$

$1 : 7$



$r_1 : r_2$

$3 : 5$

$V_1 : V_2$

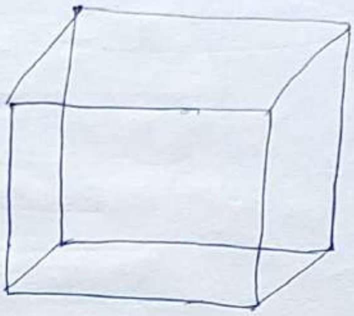
$27 : 125$

Flustum :-

$27 : 125 - 27$

$\Rightarrow \boxed{27 : 98}$

PRISM:-



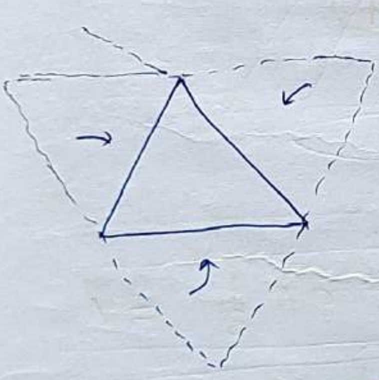
[Volume = Area of Base x hight]

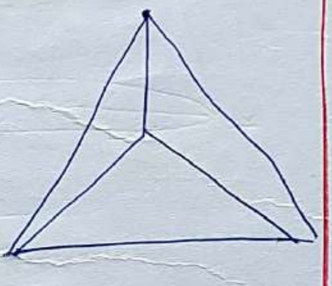
[C.S.A = Base perimeter x hight]

[T.S.A = C.S.A + 2 x B Area]

[T.S.A = B x H + 2B]

Tetrahedron :-



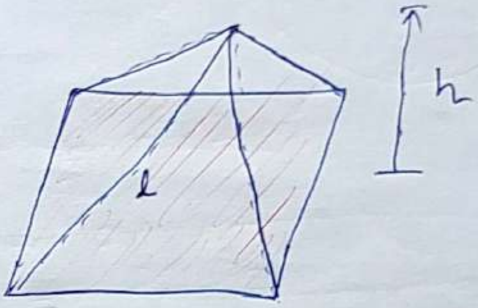


$$\boxed{V = \frac{\sqrt{2}}{12} a^3}$$

[$C.S.A = 3 \times \frac{\sqrt{3}}{4} a^2$]

[$T.S.A = 3\sqrt{3}\, a^2$]

Pyramid:-



[$V = \frac{\text{Base Area} \times \text{Hight}}{3}$]

[$C.S.A = \frac{1}{2}$ base perimet x slant]

[T.S.A = C.S.A + Base area]

Polygon :-

(1) Internal Angle + External Angle = 180

(ii) Sum of All Int. Angle = $(2n-4) \times 90^\circ$

(iii) Sum of All Ext. Angle = 360

(iv) $\boxed{\text{No of diagonals} = \frac{n(n-3)}{2}}$

Regular polygon:-

(i) Each Int. angle = $180 - \frac{360}{n}$

(ii) Each Ext angle = $\frac{360}{n}$

ALGEBRA

→ $(a+b)^2 = a^2+b^2+2ab$

→ $(a-b)^2 = a^2+b^2-2ab$

→ $a^2-b^2 = (a-b)(a+b)$

→ $\dfrac{(a+b)^2+(a-b)^2}{a^2+b^2} = 2$

→ $\dfrac{(a+b)^2-(a-b)^2}{ab} = 4$

→ $(a+b)^3 = a^3+b^3+3ab(a+b)$

→ $(a-b)^3 = a^3-b^3-3ab(a-b)$

→ $a^3+b^3 = (a+b)(a^2-ab+b^2)$

→ $a^3-b^3 = (a-b)(a^2+ab+b^2)$

→ $(a+b+c)^3 = a^3+b^3+c^3+3(a+b)(b+c)(c+a)$

→ $a^3+b^3+c^3-3abc$

$= a+b+c(a^2+b^2+c^2-ab-bc-ca)$

$= \dfrac{a+b+c}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$

Note: if $a+b+c=0$

$[a^3+b^3+c^3 = 3abc]$

Note:

$a^2+b^2+c^2-ab-bc-ca$

$= \dfrac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$

$(a+b+c)^2 = 2 \Rightarrow a^2+b^2+c^2+2(ab+bc+cd=4$

※ If $a+b+c=0$

$x+y+z=0$

then put $a=1;\ b=1;\ c=-2$

$x=1;\ y=1,\ z=-2$

Then find the solⁿ.

⇒ If $a=2,\ b=3,\ c=4$

then, $a^3+b^3+c^3-3abc = ?$

$\boxed{\text{Ans} = 9 \times \text{Middle No}}$

⇒ If $a=2 \quad b=4 \quad c=6$ (difference 2, 2)

then $a^3+b^3+c^3-3abc = ?$

⇒ $\boxed{= 9 \times \text{Middle No} \times (\text{gab})^2}$

$= 9 \times 4 \times (2)^2$

$= 36 \times 4$

⇒ If $a=2;\ b=2 \quad c=2 \pm 1$

then, $a^3+b^3+c^3-3abc =$

= Sum of a, b, c

$\boxed{\text{Ans} = a+b+c}$ (a, a, a±1)

⇒ If $a=b=c$ then

$[a^3+b^3+c^3-3abc = 0]$

Type – II

$$\frac{1}{\sqrt[3]{4}+\sqrt[3]{2}+1} = \frac{\sqrt[3]{2}-1}{2-1}$$

×2 ×2

ans (−)

$$\frac{1}{\sqrt[3]{9}+\sqrt[3]{3}+1} = \frac{\sqrt[3]{3}-1}{3-1}$$

×3 ×3

$$= \frac{\sqrt[3]{3}-1}{2}$$

Or,

$$\frac{1}{\sqrt[3]{16}-\sqrt[3]{4}+1} = \frac{\sqrt[3]{4}+1}{4+1}$$

ans (+)

$$= \frac{\sqrt[3]{4}+1}{5}$$

Type change :-

(i) $x+\frac{1}{x} = 2$

then, $x = 1$ Always

(ii) $x+\frac{1}{x} = -2$

$\boxed{x = -1}$ Always

(iii) $x+\frac{1}{x} = K$

then $\boxed{x^3+\frac{1}{x^3} = K^3-3K}$

(iv) $x-\frac{1}{x} = K$

then, $\boxed{x^3-\frac{1}{x^3} = K^3+3K}$

(v) If, $x+\frac{1}{x} = \sqrt{3}$

then, $\boxed{x^3+\frac{1}{x^3} = 0}$

$\Rightarrow \boxed{x^6 = -1}$

(vii) If $x+\frac{1}{x} = K$

$\boxed{x^2+\frac{1}{x^2} = K^2-2}$

$\Rightarrow$ If $x-\frac{1}{x} = K$

$\boxed{x^2+\frac{1}{x^2} = K^2+2}$

Note :-

$\boxed{x^2-\frac{1}{x^2} = \left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)}$

NOTE :-

$x+\frac{1}{x} = 1$

$x^2-x+1=0$

Then, $(x^3+1) = 0$ $\quad (x+1)(x^2-x+1)$

$\boxed{x^3 = -1}$

If: $\left\{x + \frac{1}{x} = K\right\}$

then

$\Rightarrow x^2 + \frac{1}{x^2} = K^2 - 2$

$\Rightarrow x^3 + \frac{1}{x^3} = K^3 - 3K$

$x^4 + \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right)^2 - 2$

$= (K^2 - 2)^2 - 2$

$$\boxed{x^5 + \frac{1}{x^5} = (K^3 - 3K)(K^2 - 2) - K}$$

$5 = 3 \times 2 - 1$

$$\boxed{5 = 3 \times 2 - 1}$$

Note:-

If $\boxed{x + \frac{1}{x} = 9}$

$$\boxed{x^5 + \frac{1}{x^5} = 9^5 - 5 \cdot 9^3 + 5 \cdot 9}$$

$$\boxed{x^5 - \frac{1}{x^5} = 9^5 + 5 \cdot 9^3 + 5 \cdot 9}$$

If $\boxed{x - \frac{1}{x} = 9}$

If $x + \frac{1}{x} = 3$ then, $x^6 + \frac{1}{x^6} = ?$

$x^3 + \frac{1}{x^3} = (K^3 - 3K) = 18$

$$\boxed{\left(x^6 + \frac{1}{x^6}\right) = (K^3 - 3K)^2 - 2}$$

$= 324 - 2$

$= 322$ Ans

Q. $x + \frac{1}{x} = 5$ then, $\frac{x^4 + 1}{x^5 + \frac{1}{x}} = ?$

Ans $\frac{x^2\left(x^2 + \frac{1}{x^2}\right)}{x^2\left(x^3 + \frac{1}{x^3}\right)}$

$= \frac{K^2 - 2}{K^3 - 3K} = \frac{25 - 2}{125 - 15}$

$= \frac{23}{110}$ Ans

If $x^2 + \frac{1}{x^2} = K$

then $\boxed{x + \frac{1}{x} = \sqrt{K + 2}}$

If $x^2 + \frac{1}{x^2} = K$

$$\boxed{x - \frac{1}{x} = \sqrt{K - 2}}$$

Q $x^2 + \frac{1}{x^2} = 11$ then, $x^3 - \frac{1}{x^3}$

$x + \frac{1}{x}$ $\Downarrow$ $\sqrt{13}$

$x - \frac{1}{x}$ $\downarrow$ $\sqrt{9} = 3$

$\Rightarrow x^3 - \frac{1}{x^3} = K^3 + 3K$

$= 27 + 9$

$= 36$ Ans

$\left\{x^2 + \frac{1}{x^2} = K\right\}$

$$\boxed{x + \frac{1}{x} = \sqrt{K + 2}} \qquad \boxed{x - \frac{1}{x} = \sqrt{K - 2}}$$

(*) If $x + \frac{1}{x} = a$

then, $\boxed{x - \frac{1}{x} = \sqrt{a^2 - 4}}$

If $x - \frac{1}{x} = a$

$\boxed{x + \frac{1}{x} = \sqrt{a^2 + 4}}$

Q. If $x + \frac{1}{x} = 4$ then,

$x^2 - \frac{1}{x^2} = ?$

Ans:-

$\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

$= 4 \times \sqrt{4^2 - 4}$

$= 4 \times \sqrt{12}$

$= 8\sqrt{3}$ A

Q. If $x - \frac{1}{x} = 3$ then,

$x^2 - \frac{1}{x^2} = ?$

$\left(x + \frac{1}{x}\right) \quad \left(x - \frac{1}{x}\right)$

$= \sqrt{3^2 + 4} \times 3$

$= \sqrt{13} \times 3 = 3\sqrt{13}$ A

V. V. I.

Q. $px + \frac{1}{qx} = a$

then, $\boxed{p^2x^2 + \frac{1}{q^2x^2} = a^2 - 2 \times \frac{p}{q}}$

$\boxed{px - \frac{1}{qx} = \sqrt{a^2 - 4 \times \frac{p}{q}}}$

$\boxed{p^3x^3 + \frac{1}{q^3x^3} = a^3 - 3a \times \frac{p}{q}}$

Q. $5x + \frac{1}{4x} = 3$ then,

$25x^2 - \frac{1}{16x^2} = ?$

A. $=$

$= \cancel{(3)^2 + 2 \times \frac{5}{4}}$

$= \cancel{9 + \frac{10}{4}}$

$\left(5x - \frac{1}{4x}\right)\left(5x + \frac{1}{4x}\right)$

$= \sqrt{(3)^2 - 4 \times \frac{5}{4}} \times 3$

$= \sqrt{4} \times 3 = 6$ A

Q. If $a = 996,\ b = 997,\ c = 998$ then,

$a^2 + b^2 + c^2 - ab - bc - ca = ?$

$= \frac{(a-b)^2 + (b-c)^2 + (c-a)^2}{2}$

$= \frac{(1)^2 + (1)^2 + (2)^2}{2}$

$= \frac{6}{2} = 3$ A

Q $(a-b)^3+(b-c)^3+(c-a)^3=?$

$x=a-b$
$y=b-c$
$z=c-a$

$(x+y+z)=0$ If

$x^3+y^3+z^3=3xyz$
$=3(a-b)(b-c)(c-a)$

Q If

$x^{1/3}+y^{1/3}\pm z^{1/3}$

then, $(x+y-z)^3+27xyz=?$

A Put $\left.\begin{array}{l} x=0 \\ y=1 \\ z=1 \end{array}\right\}$

Q If $a^2=b+c$, $b^2=c+a$, $c^2=a+b$

then, $\frac{1}{a+1}+\frac{1}{b+1}+\frac{1}{c+1}=?$

A Put $a=2;\ b=2;\ c=2$

$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$ **A**

Q If $ab+bc+ca=0$ then,

$\frac{1}{a^2-bc}+\frac{1}{b^2-ca}+\frac{1}{c^2-ab}=?$

Put, $\boxed{\begin{array}{l} a=2 \\ b=-1 \\ c=2 \end{array}}\Rightarrow$ $A=0$

Q $\frac{x^3+3x}{3x^2+1}=\frac{189}{61}$ then, $x=?$

$\frac{x^3+3x^2+3x^2+1}{x^3+3x-3x^2-1}=\frac{189+61}{189-61}$

Note:

$\frac{a}{b}=\frac{c}{d}$

$\boxed{\frac{a+b}{a-b}=\frac{c+d}{c-d}}$ Componendo Dividendo Rule

$\frac{(x+1)^3}{(x-1)^3}=\frac{250}{128}=\frac{125}{64}$

$\frac{x+1}{x-1}=\frac{5}{4}\Rightarrow\boxed{x=9}$ **A**

Q $(a-1)^2+(b+2)^2+(c-2)^2=0$

then, $a+b+c=?$

$a=1$, $b=-2$, $c=2$

$a+b+c=1$ **A**

Q Imp

$a^2+b^2+c^2=2(a-b-c)-3$

then, $2a+3b+4c=?$

A

$a^2+b^2+c^2=2(a-b-c)-3$

1 -1 -1 तीन बराबर टुकड़े

$a=1$
$b=-1$
$c=-1$

$2a+3b+4c=-5$ **A**

method

$a^2+b^2+c^2-2a+2b+2c+3=0$

$(a-1)^2+(b+1)^2+(c+1)^2=0$

$a=1$, $b=-1$, $c=-1$

Q $a^x = (x+y+z)^y$

$a^y = (x+y+z)^z$

$a^z = (x+y+z)^x$

then, $x+y+z = ?$

$xyz = ?$

A

$a^{x+y+z} = (x+y+z)^{x+y+z}$

$\boxed{a = x+y+z}$

$x = \frac{a}{3}, \quad y = \frac{a}{3}, \quad z = \frac{a}{3}$

$\boxed{xyz = \frac{a^3}{27}}$

Q $\frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} + \frac{d}{1-d} = 1$

then, $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} + \frac{1}{1-d} = ?$

Ans:-

$\frac{a}{1-a}+1 + \frac{b}{1-b}+1 \ldots = 1+4$

$= 5$ A

$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} + \frac{1}{1-d} = 5.$

Q $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} = 1$

th $\frac{a^2}{b+c} + \frac{b^2}{c+a} + \frac{c^2}{a+b} = ?$

Ans:- 0.

Q If $x = 2^{1/3} + 2^{-1/3}$

then, $2x^3 - 6x = ?$

A.

$\cancel{x(2^{1/3}+2^{-1/3})} - 6($

$x^3 = (2^{1/3} + 2^{-1/3})^3$

$= (2^{1/3})^3 + (2^{-1/3})^3 + 3\,2^{1/3}\,2^{-1/3}(x)$

$x^3 = 2 + \frac{1}{2} + 3x$

$(x^3 - 3x) = \frac{5}{2}$

$\boxed{2x^3 - 6x = 5}$

Trick:-

$x = 2^{1/3} + 2^{-1/3}$

base

then $2x^3 - 6x$

$\boxed{\text{base} - 3\times b}$

$\boxed{\text{Sol}^n = \text{base}^2 + 1}$

Q. $x = 5^{1/3} + 5^{-1/3}$ then,

b

$5x^3 - 15x + 7 = ?$

b $\quad$ $3\times b$

$\Rightarrow = b^2 + 1 + 7$

$= 26 + 7 = 33$ A

* If base digit comes 3 times:-

Q. $x = 2 - 2^{1/3} + 2^{2/3}$

2 = base

then, $x^3 - 6x^2 + 18x = ?$

$\downarrow$ 1 $\quad \downarrow$ $3 \times b$ $\quad \downarrow$ $3b \times (b+1)$

$$\boxed{\text{sol}^n:- = b^3 + 4b^2 - b}$$

A $= 8 + 16 - 2$

$= 22$ A

Eg: $x = 5 - 5^{1/3} + 5^{1/3}$

then $x^3 - 15x^2 + 90x + 10$

$= b^3 + 4b^2 - 5 \quad + 60$

$= 125 + 100 - 5 \quad + 10$

$= 285 - 5$

$= 280$ A

Q. $x = 3 - 3^{1/3} + 3^{2/3}$

then $2x^3 - 18x^2 + 72x + 12 = ?$

$2(x^3 - 9x^2 + 31x) + 12$

$= 2 \times (b^3 + 4b^2 - b) + 12$

$= 2(27 + 36 - 3) + 12$

$= 120 + 12 = 132$ A

Note:-

If $x + \frac{1}{x} = 0$

then, $x^{18} + x^{12} + x^6 + 1 = 0$

$x + \frac{1}{x} = 0 \Rightarrow (x^2 = -1)$

$= (x^2)^9 + (x^2)^6 + (x^2)^3 + 1$

$= (-1)^9 + (-1)^6 + (-1)^3 + 1$

$= -1 + 1 - 1 + 1 = 0$ A

Trick:- Cut method.
एक को दूसरे से काटो

$\cancel{x^{18}} + \cancel{x^{12}} + \cancel{x^6} + \cancel{1} = 0$

एक जरूर होगा

(ii) $\left(x + \frac{1}{x}\right)^2 = 3$

$x + \frac{1}{x} = \sqrt{3}$

$x^6 = -1$

then, $\cancel{x^{206}} + x^{200} + \cancel{x^{90}} + \cancel{x^{84}} + x^{18} + x^{12} + 1$

$= 0 + 0 + 0 + 1 = 1$

(iii) $x + \frac{1}{x} = 0$

$x^2 = -1$

$x^{20} + x^{16} + x^{12} + x^8 + x^4 + 1$

$= 1 + 1 + 1 + 1 + 1 + 1$

$= 6$ A

Note:-

→ If diff = 3, 2, 6 तो
एक दूसरे को काटो

→ If diff = 4 तो
एक दूसरे को जोड़ो (1) मानकर

→ If there is any other value at the end let say 6 then make it $\underline{\underline{1+5}}$.

→ Last में 1 जरूर होगा।

(*)

If $x = 16$

then

$x^4 - 17x^3 + 17x^2 - 17x + 17 = ?$

$= x^4 - (16+1)x^3 + (16+1)x^2 - (16+1)x + 17$

$= \cancel{x^4} - \cancel{x^4} - \cancel{x^3} + \cancel{x^3} + \cancel{x^2} - \cancel{x^2} (-x + 17)$

$= -x + 17$

$= -16 + 17 = 1$ Ans

$(-16+17)$

$a+b = 11$ ab
$a^2+b^2 = 51$
$121 = 51 + 2ab$ (35)
$b^2+b^2+c^2$
$-2bc+c^2$ $a+b = 11$ a^2+
$2(b^2+c^2)-2bc$ $\cancel{ab = 35}$

$\frac{121}{51}$
$b+c = 11$
$bc = 35$

Q If
$a+b+c = 11$
$ab+bc+ca = 35$
then
$(a-b)^2 + (b-c)^2 + (c-a)^2 = ?$

A Put
$a = 5$
$b = 5$
$c = 1$

$c = 0$
$a+b = 11$
$ab = 35$
$a^2+b^2-2ab+b^2+a^2$
$2\{(a+b)^2-2ab\}-2ab$
$= 2(a+b)^2 - 5ab$ 140
$= 242 - 175$

Q If $a+b+c = 11$

$a^2+b^2+c^2 = 51$ then

$ab+bc+ca = ?$

Put $a = 5$, $b = 5$, $c = 1$ Ans 35

Put $c = 0$

(*) Min values :-

Find Min. value of $x^2 - x + 1$

A $\frac{d}{dx}(x^2 - x + 1) = 0$

$\Rightarrow 2x - 1 = 0$

$x = \frac{1}{2}$

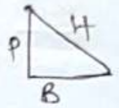
Put in eqn

$= \frac{1}{4} - \frac{1}{2} + 1$

$= -\frac{1}{4} + 1 = \frac{3}{4}$ Ans

$x^3 + 1^3 = (1-1)$

Q $3\sin\theta + 4\cos\theta = 5$
$\downarrow P \quad \downarrow B \quad \downarrow H$



$$\left[\tan\theta = \frac{P}{B} = \frac{3}{4}\right]$$

$$\left[(a^2-b^2)\sin\theta + ab\cos\theta = a^2+b^2\right]$$
$\downarrow P \quad \downarrow B \quad H$

⇒ Q $10\sin^4\theta + 15\cos^4\theta = 6$

Find $27\csc^6\theta + 8\sec^6\theta = ?$

⇒

$$10\sin^4\theta + 15\cos^4\theta = 6$$

$$\frac{10}{6}\sin^4\theta + \frac{15}{6}\cos^4\theta = 1$$

$(\text{But } \sin^2\theta + \cos^2\theta = 1)$

$$\therefore \frac{10}{6} = \frac{1}{\sin^2\theta} = \frac{5}{3} = \csc^2\theta$$

$$\frac{15}{6} = \frac{1}{\cos^2\theta} = \frac{5}{2} = \sec^2\theta$$

$\therefore 27\csc^6\theta + 8\sec^6\theta$

$= \cancel{27} \times \frac{125}{\cancel{27}} + \cancel{8} \times \frac{125}{\cancel{8}}$

$= 250$ Ans

Quadratic Equation:-

$$ax^2 + bx + c = 0$$

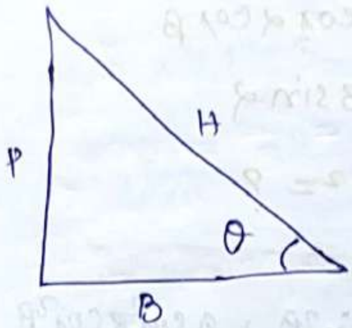
$$\boxed{\begin{aligned} \alpha + \beta &= \frac{-b}{a} \\ \alpha\beta &= \frac{c}{a} \end{aligned}}$$

Quadratic eqn can be formed

$\boxed{x^2 - (\text{sum of root})x + \text{product of Root} =}$

(*) $ax^3 + bx^2 + cx + d = 0$

$$\boxed{\begin{aligned} \alpha + \beta + \gamma &= -\frac{b}{a} \\ \alpha\beta + \beta\gamma + \gamma\alpha &= \frac{+c}{a} \\ \alpha\beta\gamma &= -\frac{d}{a} \end{aligned}}$$



$\sin\theta = \frac{P}{H} = \frac{p}{h}$, $\operatorname{cosec}\theta = \frac{H}{P}$

$\cos\theta = \frac{B}{H} = \frac{b}{h}$ $\sec\theta = \frac{H}{B}$

$\tan\theta = \frac{P}{B} = \frac{p}{b}$ $\cot\theta = \frac{B}{P}$

(*) If any side out of P or B is greater than 1 then ratio of sides:-

$$\boxed{a : \frac{a^2-1}{2} : \frac{a^2+1}{2}}$$

3	4	5
5	12	13
7	24	25
9	40	41
8	15	17

If, even no then,

$$\boxed{a : \left(\frac{a}{2}\right)^2 - 1 : \left(\frac{a}{2}\right)^2 + 1}$$

$\Rightarrow$ 8 : 15 : 17
12 16 20

Value of Trigonometric Identity on diff

	0°	30°	45°	60	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

Note:- If $x + y = 90^\circ$
or, $\theta_1 + \theta_2 = 90$

$$\begin{bmatrix} \sin\theta_1 = \cos\theta_2 \\ \tan\theta_1 = \cot\theta_2 \\ \operatorname{cosec}\theta_1 = \sec\theta_2 \end{bmatrix}$$

or,

$\frac{\sin\theta_1}{\cos\theta_2} = 1$

$\frac{\tan\theta_1}{\cot\theta_2} = 1$

$\frac{\operatorname{cosec}\theta_1}{\sec\theta_2} = 1$

(*) $\sin 3\theta = \cos(\theta - 26)$ then $\theta = ?$

$3\theta + \theta - 26 = 90$

$4\theta = 116$

$\boxed{\theta = 29}$

Note:-

$\boxed{\sin^2\theta + \cos^2\theta = 1}$

$\boxed{\sec^2\theta - \tan^2\theta = 1}$

$\boxed{\operatorname{cosec}^2\theta - \cot^2\theta = 1}$

If $x + y = 90$ then,

$\sin^2 x + \sin^2 y = 1$

$\cos^2 x + \cos^2 y = 1$

eg:-

$\sin^2 1 + \sin^2 2 + \sin^2 88 + \sin^2 89$

$= 1 + 1$

$= 2$

Q $\sin^2 1 + \sin^2 2 + \cdots + \sin^2 88 + \sin^2 89$

$= 44 + \left(\frac{1}{\sqrt{2}}\right)^2$

$= 44 + \frac{1}{2} = 44\frac{1}{2}$

or, $\frac{\text{No. of term}}{2}$

$= \frac{N}{2} = \frac{89}{2}$

$= 44\frac{1}{2}$ Ans

Q. $(\sin^2 1 + \sin^2 2 + \cdots + \sin^2 88 + \sin^2 89) + \sin^2 90$

$= \frac{N}{2} + (1)^2$

$= 44\frac{1}{2} + 1$

$= 45\frac{1}{2}$

(Q) $\sin^2 1 + \sin^2 5 + \sin^2 9 \cdots + \sin^2 89$

$N = \frac{89-1}{4} + 1$

$= \frac{88}{4} + 1 = 23$

Ans:- $\frac{N}{2} = \frac{23}{2} = 11\frac{1}{2}$ Ans

(*) If $\sin x + \sin^2 x = 1$

then $\boxed{\cos^2 x + \cos^4 x = 1}$

If $\cos x + \cos^2 x = 1$

$\boxed{\sin^2 x + \sin^4 x = 1}$

Q) If $x = 3\cos\alpha \sin\beta$

$y = 3\cos\alpha\cos\beta$

$z = 3\sin\alpha$

$x^2 + y^2 + z^2 = ?$

A $9\cos^2\alpha\sin^2\beta + 9\cos^2\alpha\cos^2\beta + 9\sin^2\alpha$

$= 9\cos^2\alpha(\sin^2\beta + \cos^2\beta) + 9\sin^2\alpha$

$= 9\cos^2\alpha + 9\sin^2\alpha$

$= 9$ A

(*) Note:-

$\sin^2\theta + \cos^2\theta = 1$

$\frac{a}{b}\sin^2\theta + \frac{c}{d}\cos^2\theta = 1$

then, $[a = b]$; $[c = d]$

$\Rightarrow$ $a\sin\theta + b\cos\theta = 1$

$[a = \sin\theta]$; $[b = \cos\theta]$

Q $\cos^2\theta - \sin^2\theta = 1/3$

then, $\cos^4\theta - \sin^4\theta + 1 = ?$

A $(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta) + 1$

$\Rightarrow 1 \times \left(\frac{1}{3}\right) + 1$

$= 4/3$ Ans

Q $x\sin^3\alpha + y\cos^3\alpha = \sin\alpha\cos\alpha \neq 0$

then $x^2+y^2 = ?$

Ans $\frac{x\sin^2\alpha}{\cos\alpha} + \frac{y\cos^2\alpha}{\sin\alpha} = 1$

we know, $\sin^2\alpha + \cos^2\alpha = 1$

$x = \cos\alpha, \quad y = \sin\alpha$

$\boxed{x^2+y^2 = 1}$

(*) **Note:-** $x+y = 90°$ then,

$\boxed{\tan x = \cot y}$

$\boxed{\tan x \tan y = 1}$

$\boxed{\cot x \cot y = 1}$

A, $\boxed{\sin x \sec y = 1}$

$\sin x = \cos y$

Q $\tan 1 \times \tan 4 \times \tan 45 \times \tan 86 \times \tan 89$

$= 1 \times 1 \times 1 = 1$

Q $\tan 1 \tan 2 \tan 3 \cdots \tan 89° = ?$

A $= \underline{1}$

(*) **Note:-**

$\sin\theta + \operatorname{cosec}\theta = 2$

means $\sin\theta + \frac{1}{\sin\theta} = 2$

$\therefore \sin\theta = 1$ always

$\therefore \sin^{100}\theta + \frac{1}{\sin^2\theta} = 1+1 = 2$

(ii) $\cos\theta + \sec\theta = 2$

$\cos\theta + \frac{1}{\cos\theta} = 2$

$\downarrow$

$\cos\theta = 1$ always

Note:-

$\sec^2\theta - \tan^2\theta = 1$

$\downarrow$

$(\sec\theta + \tan\theta)(\sec - \tan\theta) = 1$

Means:

$\left.\begin{array}{l}\sec\theta + \tan\theta = a \\ \sec\theta - \tan\theta = 1/a\end{array}\right\}$

Or, $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$(\operatorname{cosec}\theta - \cot\theta)(\operatorname{cosec}\theta + \cot\theta) = 1$

$\downarrow$

If, $\left.\begin{array}{l}\operatorname{cosec}\theta - \cot\theta = a \\ \operatorname{cosec}\theta + \cot\theta = 1/a\end{array}\right\}$

Q $\sec^2\theta + \tan^2\theta = 2$

then, $\sec^4\theta - \tan^4\theta + 1 = ?$

Ans $(\sec^2\theta - \tan^2\theta)(\sec^2\theta + \tan^2\theta) + 1$

$= 1 \times 2 + 1 = 3$

Imp

Note:- If $a\sin\theta + b\cos\theta = c$

then $a\cos\theta - b\sin\theta = ?$ (x)

$\boxed{= \pm\sqrt{a^2+b^2-c^2}}$ Square and add

and, $a\cos\theta - b\sin\theta = c$

$a\sin\theta + b\cos\theta = ?$

$= \pm\sqrt{a^2+b^2-c^2}$ Ans

Q If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$

then, $\cos\theta - \sin\theta = ?$

A $a = 1 \quad b = 1 \quad c = \sqrt{2}\cos\theta$

$\therefore \cos\theta - \sin\theta = \sqrt{a^2 + b^2 - c^2}$

$= \sqrt{1 + 1 - 2\cos^2\theta}$

$= \sqrt{2(1 - \cos^2\theta)}$

$= \sqrt{2}\sin\theta$ A

(*) Must Remember:-

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

Now

(i) $\sin(15^\circ) = \sin(45^\circ - 30^\circ)$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

$\boxed{\cos 75^\circ = \sin 15^\circ = \dfrac{\sqrt{3}-1}{2\sqrt{2}}}$

(ii) $\sin 75^\circ = \cos 15^\circ$

$= \sin(45 + \sin 30)$

$\boxed{\sin 75^\circ = \dfrac{\sqrt{3}+1}{2\sqrt{2}} = \cos 15^\circ}$

(iii) $\tan 15^\circ = \tan(45 - 30^\circ)$

$= \dfrac{\tan 45 - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$

$= \dfrac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

$= 2 - \sqrt{3}$

$\boxed{\tan 15^\circ = \cot 75^\circ = 2 - \sqrt{3}}$

(iv) $\tan 75^\circ = \cancel{\tan}\ \tan(45 + 30^\circ)$

$= \dfrac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$

$= \dfrac{\sqrt{3}+1}{\sqrt{3}-1} = \dfrac{(\sqrt{3}+1)^2}{2}$

$= 2 + \sqrt{3}$

$\boxed{\tan 75^\circ = \cot 15^\circ = 2 + \sqrt{3}}$

Nutshell:-

(i) $\sin 15^\circ = \cos 75^\circ = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

(ii) $\sin 75^\circ = \cos 15^\circ = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

(iii) $\tan 15^\circ = \cot 75^\circ = 2 - \sqrt{3}$

(iv) $\tan 75^\circ = \cot 15^\circ = 2 + \sqrt{3}$

Q $x = \tan 75^\circ$

~~then~~ $\left(x + \frac{1}{x}\right)^2 = ?$

A

$= \tan 75^\circ + \dfrac{1}{\tan 75^\circ}$

$= \tan 75^\circ + \cot 75^\circ$

$= 2 + \sqrt{3} + 2 - \sqrt{3}$

$= (4)^2$ A $= 16$ A

Q If

$(1-\sin\alpha)(1-\sin\beta)(1-\sin\gamma)$

$= (1+\sin\alpha)(1+\sin\beta)(1+\sin\gamma)$

$= \pm \cos\alpha \cos\beta \cos\gamma$

Q If $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$

then $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 = ?$

$= 0$ Ans

$\theta_1 = \theta_2 = \theta_3 = \underline{90°}$

Note:-

$\boxed{\sin\theta \sin(60-\theta)\sin(60+\theta) = \frac{1}{4}\sin 3\theta}$

$\boxed{\cos\theta \cos(60-\theta)\cos(60+\theta) = \frac{1}{4}\cos 3\theta}$

$\boxed{\tan\theta \cdot \tan(60-\theta)\tan(60+\theta) = \tan 3\theta}$

Q $\cos 20 \cos 40 \cos 60 \cos 80 = ?$

$\Rightarrow \cos 20 \cos 40 \cos 80 \times \cos 60 =$

$= \cos\theta \cos(60-\theta)\cos(60+\theta) \times \cos 60$

$= \frac{1}{4} \times \cos 60 \times \cos 60$

$= \frac{1}{16}$ Ans

Q $\cos 10 \cos 20 \cos 40 = ?$

$= \frac{\cos 10 \cos 20 \cos 40 \times \cos 80}{\cos 80}$

$= \frac{\cos 10}{\cos 80}\left[\frac{1}{4} \times \frac{1}{2}\right]$

$= \frac{\cos 10}{\sin 10}\left[\frac{1}{8}\right]$

$= \frac{\cot 10}{8}$ Ans

Q $\tan 20 \tan 40 \tan 60 \tan 80 = ?$

A $\underbrace{\tan 20 \tan 40 \tan 80}_{\tan 60} \underbrace{\tan 60}_{\tan 60}$

$= \sqrt{3} \times \sqrt{3} = 3$ Ans

Q $x = a\sec^n\theta, \quad y = b\tan^n\theta$

then,

$\sec^n\theta = \left(\frac{x}{a}\right)$

$\sec^2\theta = \left(\frac{x}{a}\right)^{\frac{2}{x}}, \quad \tan^2\theta = \left(\frac{y}{b}\right)^{\frac{2}{n}}$

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \boxed{\left(\frac{x}{a}\right)^{\frac{2}{x}} - \left(\frac{y}{b}\right)^{\frac{2}{n}} = 1}$

Q If $x = \cos t, \quad y = 2\sin t \cos t$

$\sin t = \frac{y}{2x}$

$= \frac{y}{2x}$

$\sin^2 t + \cos^2 t = 1$

$\Rightarrow \frac{y^2}{4x^2} + x^2 = 1$

$\Rightarrow [y^2 = 4x^2 - 4x^4]$

$\boxed{\sin 2\theta = 2\sin\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}}$

$\boxed{\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta}$

$\boxed{= \frac{1-\tan^2\theta}{1+\tan^2\theta}}$

$\boxed{\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}}$

Q $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \frac{1+\cos\theta}{\sin\theta}$

$= \frac{\cancel{1}+(2\cos^2\frac{\theta}{2})-\cancel{1}}{2\sin\theta/2 \, \cos\theta/2}$

$= \cot\frac{\theta}{2}$ Ans

Q $\sqrt{2+2\cos 2\theta} = ?$ $2\cos\theta$

$\sqrt{2+2(2\cos^2\theta-1)}$

$= \boxed{2\cos\theta}$

Q $\sqrt{2+\sqrt{2+2\cos 4\theta}}$

$= 2\cos\left(4\theta\times\frac{1}{2}\times\frac{1}{2}\right)$

$= 2\cos\theta$

Q $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}$

3 Roots

$= 2\cos\left(8\theta\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)$

$= 2\cos\theta$

Q $\sqrt{2+2\cos x}$ $\quad$ $\sqrt{2+2\cos 7\theta}$

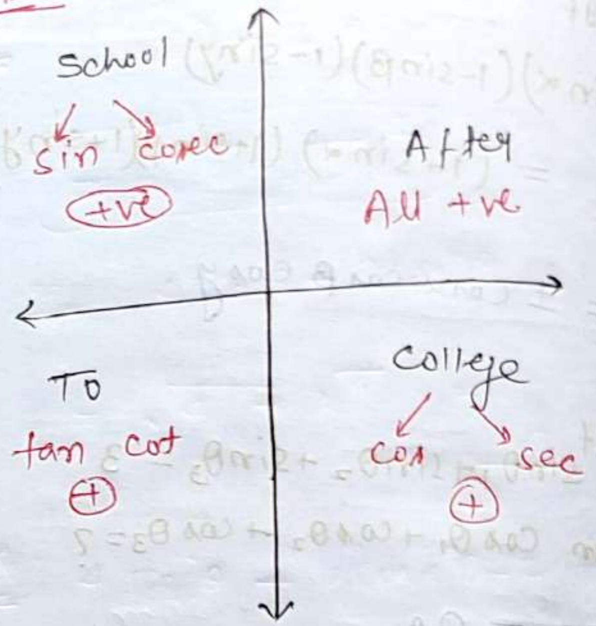
$2\cos\frac{x}{2}$ $\quad$ $2\cos\frac{7\theta}{2}$

Q $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 7\theta}}}$

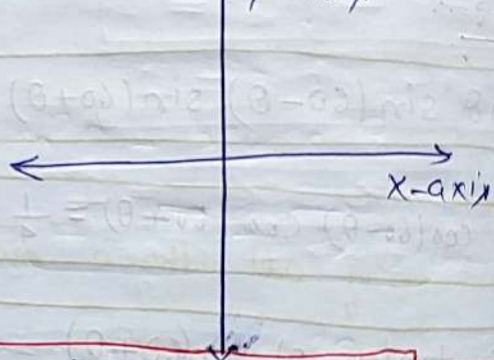
$= 2\cos\left(7\theta\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)$

$= 2\cos\left(\frac{7\theta}{8}\right)$ Ans

Note:-



Change Rule :-



X-axis = No change (0, 180 ±)

Y-axis = change (90, 270 ±)

Eg :- $\sin(300) = \sin(360-60)$

$= -\sin(-60)$

$= -\sin 60$

$\sin(300) = \sin(270+30)$

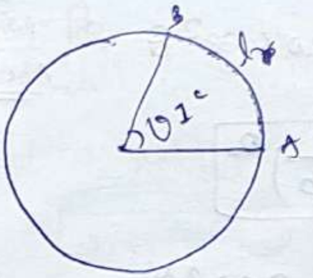
according to it sign will change

$= -\cos(30)$

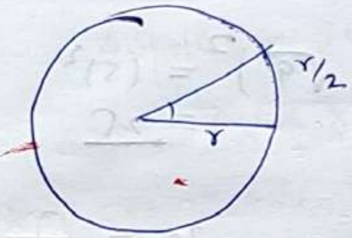
IV quadrant

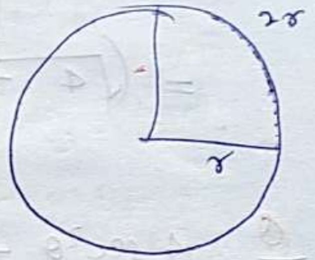
$= -\frac{\sqrt{3}}{2}$ Ans

4th quadrant sin ⇒ -ve so -ve sign



$$\boxed{\theta = \frac{l}{r}}$$





$$\theta^c = \frac{r/2}{r}$$

$$\boxed{\theta = \left(\frac{1}{2}\right)^c}$$

$$\theta^c = \frac{2r}{r}$$

$$\boxed{\theta = 2^c}$$

$$\boxed{1^c = 57^\circ 16' 22''}$$

16+6

Note:-

Sum of all three sides of Triangle $= \pi^c$

$$\boxed{A^c + B^c + C^c = \pi^c = \frac{22}{7}}$$

(*)

$$\boxed{\begin{aligned} \text{Radian} \times \frac{180}{\pi} &= \text{degree} \\ \text{degree} \times \frac{\pi}{180} &= \text{Radian} \end{aligned}}$$

Q. $A = \sin 1^c - \sin 57^\circ$ then

$A > 0$, $A < 0$, $A = 0$

A. $\boxed{1^c = 57^\circ 16' 22''}$

$1^c > 57^\circ$

$\therefore A > 0$ Ans

* Minimum and Maximum Value

(I) $\sin\theta$: Max 1, Min -1

$\cos\theta$: Max 1, Min -1

(II) $\tan\theta$, $\cot\theta$, $\sec\theta$, $\csc\theta$,

$\Rightarrow$ Max^m value = can't find

$\Rightarrow$ Minimum value = can be find.

Q. $a\sin\theta + b\cos\theta$, max/min $\Rightarrow$?

$$\text{Max} = \sqrt{a^2+b^2}$$

$$\text{Min} = -\sqrt{a^2+b^2}$$

Eg:- $3\sin\theta + 4\cos\theta$

$$\text{Max} = \sqrt{3^2+4^2} = 5$$

$$\text{Min} = -\sqrt{3^2+4^2} = -5$$

Q. $8^{\sin\theta} \cdot 16^{\cos\theta}$ find Max, Min

A. $2^{3\sin\theta} \cdot 2^{4\cos\theta}$

$$= 2^{(3\sin\theta + 4\cos\theta)}$$

$3\sin\theta + 4\cos\theta =$ Max $= 5$, Min $= -5$

$$\Rightarrow \text{Max} = 2^5$$

$$\text{Min} = 2^{-5} = \frac{1}{25} \text{ Ans}$$

Q $a\sin^2\theta + b\cos^2\theta$, Max Min

	Max	Min
If $a>b$	a	b
If $b>a$	b	a

Eg:- $2\sin^2\theta + 3\cos^2\theta$

Max $= 3$, Min $= 2$

Q $\sin^8\theta + \cos^{14}\theta$ then max^m value.

Ans 1 $\left[\begin{matrix}\theta = 0^\circ \\ \theta = 90^\circ\end{matrix}\right]$

If n = even both

Q If $A = \sin\theta\cos\theta$ then A?

$A = \dfrac{2\sin\theta\cos\theta}{2}$

$A = \dfrac{\sin 2\theta}{2}$

$\left[-\frac{1}{2} \le A \le \frac{1}{2}\right]$

or, $\theta = 45^\circ$

Q If $A = (\sin^2\theta\cos^2\theta)$

$A = \sin^2\theta(1-\sin^2\theta)$

$\theta = 0^\circ$, $\theta = 45^\circ$

$0 \le A \le \frac{1}{4}$

or $(\sin\theta\cos\theta)^2$

$= \left(\dfrac{\sin 2\theta}{2}\right)^2$

$0 \le A \le \frac{1}{4}$

Q $a\sec^2\theta + b\operatorname{cosec}^2\theta$ की Min Answer

$\boxed{= (\sqrt{a}+\sqrt{b})^2}$

Eg:- $4\sec^2\theta + 9\cos^2\theta$

$= (\sqrt{4}+\sqrt{9})^2 = (5)^2$
$= 25$

Q $4\sec^2\theta + 9\operatorname{cosec}^2\theta - 7$ की Min value

$= (\sqrt{4}+\sqrt{9})^2 - 7$

$= 25 - 7 = 18$ Ans

Q $a\tan^2\theta + b\operatorname{cosec}^2\theta$

Min $\boxed{= (\sqrt{a}+\sqrt{b})^2 - a}$

$a(\sec^2\theta - 1) + b\operatorname{cosec}^2\theta$

$\Rightarrow a\sec^2\theta + b\operatorname{cosec}^2\theta - a$

$\Rightarrow (\sqrt{a}+\sqrt{b})^2 - a$ Ans

Eg:- $4\tan^2\theta + 9\operatorname{cosec}^2\theta$

Min $= (\sqrt{4}+\sqrt{9})^2 - 4$

$= 25 - 4 = 21$ Ans

Q $a\sec^2\theta + b\cot^2\theta = $?

$\boxed{\text{min} = (\sqrt{a}+\sqrt{b})^2 - b}$

Eg: $4\sec^2\theta + 9\cot^2\theta$

$= (\sqrt{2}+\sqrt{9})^2 - 9$

$= 25 - 9$

$= 16$ Ans

(*) $a\tan^2\theta + b\cot^2\theta =$

Min value $= (\sqrt{a} + \sqrt{b})^2 - a - b$

$$\boxed{\text{Min} = 2\sqrt{ab}}$$

Eg: $4\tan^2\theta + 9\cot^2\theta$

$= 2\sqrt{4\times 9} = 12$ Ans

Now,

Min

$a\tan^2\theta + b\cot^2\theta = 2\sqrt{ab}$

$a\sin^2\theta + b\cos^2\theta = 2\sqrt{ab}$ $(a>b)$

$a\cos^2\theta + b\sec^2\theta = 2\sqrt{ab}$

→ Reciprocal के case में

Eg:- $\cos^2\theta + 3\sec^2\theta$. Min value

Min $= 2\sqrt{1\times 3}$

$= 2\sqrt{3}$ Ans

If $a<b$
min $= a+b$

Q $\sin^2\theta + \cos^2\theta + (\tan^2\theta + \cot^2\theta) + \sec^2\theta + \operatorname{cosec}^2\theta$

↓

Min

Ans $1 + 2\sqrt{1\times 1} + (\sqrt{1}+\sqrt{1})^2$

$= 1 + 2 + 4 = 7$ Ans

Q

$= \sin^{2m}\theta + \cos^{2m}\theta \le 1$

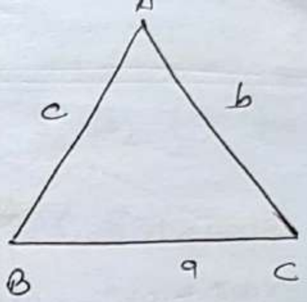
~~Min~~

Max $\begin{cases} 1 & \theta = 0 \\ & \theta = 90^\circ \\ \text{Min} & \text{Put } \theta = 45^\circ \end{cases}$

Q

$\sin^n\theta \cos^n\theta = \left\{\frac{1}{2^n}\right\}$

(*)

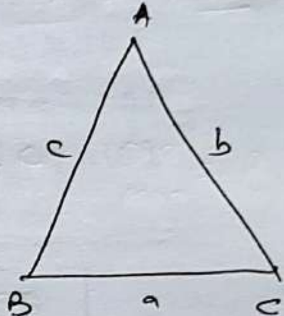


Area of $\Delta = \frac{1}{2}ab\sin C$

$= \frac{1}{2}ac\sin B$

$= \frac{1}{2}cb\sin A$

Sine Rule:-



$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

or, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule:- $a \longrightarrow b \longrightarrow c$

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos B = \frac{c^2 + a^2 - b^2}{2ca}$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

Q If $\frac{a}{\cos A} = \frac{b}{\cos B} = \frac{c}{\cos C}$

then ΔABC

Ans (Equilateral Triangle)

Number System

Basic Things :-

$\boxed{0+1+2+3+4+\cdots N = \frac{n(n-1)}{2}}$

$\boxed{1+2+3+4+5+6+\cdots N = \frac{n(n+1)}{2}}$

$\boxed{1+3+5+7+\cdots+N = n^2}$

$\boxed{2+4+6+8+\cdots+N = n(n+1)}$

$\boxed{1^2+2^2+3^2+\cdots+N^2 = \frac{n(n+1)(2n+1)}{6}}$

$\boxed{1^3+2^3+3^3+\cdots+N^3 = \left\{\frac{n(n+1)}{2}\right\}^2}$

n = No. of terms

$[\,1^2+3^2+5^2+7^2+\cdots+N^2 = \frac{N(N+1)(N+2)}{6}$

$[\,2^2+4^2+6^2+8^2+\cdots+N^2 = \frac{N(N+1)(N+2)}{6}$

$\boxed{N \to \text{Last term}}$

$n =$

Airethmetic Progression :-

$n_1 \quad n_2 \quad n_3 \quad n_4 \cdots = n$

$\Rightarrow a \quad a+d \quad a+2d \quad a+3d \quad \cdots a+(n-1)d$

$\Rightarrow \boxed{T_n = a+(n-1)d}$

$\Rightarrow \boxed{S_n = \frac{n}{2}\left[2a+(n-1)d\right]}$

$= \frac{n}{2}\left[a + l\right]$ ✓

1st term, last term

$\Rightarrow \boxed{\text{no. of term} = \frac{l-a}{d}+1}$

$1^3+3^3+5^3+7^3+9^3 \cdots\cdots N.$

$\Rightarrow n^2(2n^2-1)$

Prime NO :- 2, 3, 5, 7, 9

Composite NO :- 0, 4, 6, 8 ...

1 → is neither prime nor composite

✱ ⇒ A no. of type $\boxed{abab}$ is always divisible by **101**.

Eg :-

5050 = 50 × 101

2525 = 25 × 101

1717 = 17 × 101

Note :-

$1+2+3+\cdots+10 = 55$

$1+2+3+\cdots+100 = 5050$

⇒ A no. of type $\boxed{ababab}$ is always divisible by $\boxed{10101}$

Eg :-

101010 = 10 × 10101

151515 = 15 × 10101

$\boxed{abcabc \Rightarrow 1001 \text{ से divisible}}$

Q. $1+2+3+\cdots+100+99+\cdots+1 = ?$

$\Rightarrow 5050+5050-100 = 10{,}000$

Or, $(100)^2 = 10000$

Trick :-

$1+2+3+\cdots+n+(n-1)+(n-2)+\cdots 1$

Ans $= n^2$

Q $41 + 42 + \cdots + 100 = ?$

$a \qquad l$

$$\Rightarrow S_n = \left(\frac{a+l}{2}\right) \times n$$

$$= \frac{100+41}{2} \times 60$$

$$= 141 \times 30$$

$$= 4230 \quad A$$

Q. $1^2 + 2^2 + 3^2 + \cdots + 10^2 = ?$

$$= \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{10 \times 11 \times 21}{6} = 35 \times 11$$

$$= 385$$

Imp.

Q If

$$1^2 + 2^2 + 3^2 + \cdots + 10^2 = 385$$

then, ($\times 2$)

$$2^2 + 4^2 + 6^2 + \cdots + 20^2 = ?$$

A

$$2^2 (1^2 + 2^2 + 3^2 + \cdots + 10^2) =$$

$$= 2^2 \times 385$$

$$= 4 \times 385 = 1540$$

and, ($\times 3$)

$$3^2 + 6^2 + 9^2 + \cdots + 30^2 = ?$$

$$3^2 (1^2 + 2^2 + 3^2 + \cdots + 10^2) =$$

$$= 9 \times 385$$

$$= 3465$$

Q $1^2 - 2^2 + 3^2 - 4^2 + 5^2 + \cdots - 10^2 = ?$

$$1^2 + 3^2 + 5^2 + \cdots + 9^2 = \frac{n(n+1)(n+2)}{6}$$

$$= \frac{5(5+1)(5+2)}{6}$$

$$\boxed{n = 9} \quad = \frac{9 \times 10 + 11}{6}$$

$$= 165$$

$$2^2 + 4^2 + 6^2 + 8^2 + 10^2 = \frac{n(n+1)(n+2)}{6}$$

$$= \frac{10 \times 11 \times 12}{6}$$

$$= 220$$

$$\text{Ans} = 165 - 220 = -55$$

Trick:-

(*) $1^2 - 2^2 + 3^2 - 4^2 + 5^2 + \cdots 9^2 - 10^2 = ?$

Take last sign :-

$$= -(1 + 2 + 3 + 4 + \cdots + 10)$$

$$= -55$$

(ii) $1^2 - 2^2 + 3^2 - 4^2 + 5^2 + \cdots - 99^2 + 100^2 = ?$

$$+(1 + 2 + 3 + 4 + \cdots + 100)$$

$$= 5050 \quad A$$

(*) From 1 to 100 :-

2 to 9 ⇒ 20 times

1 ⇒ 21 times

0 ⇒ 11 times

(*) Digit requires to write page no 1 to 100 in a book :-

1 − 9 ⇒ 9 × 1 = 9

10 − 99 ⇒ 90 × 2 = 180

100 ⇒ = 3

= 192 Ans

(*)

$$\boxed{\begin{array}{l} \text{No of Hand shake} = \dfrac{n(n-1)}{2} \\ \text{No. of gift distributed} = n(n-1) \end{array}}$$

Q. $1\times2 + 2\times3 + 3\times4 + \cdots 10\times11 = ?$

$1\times2 = 1^2+1$

$2\times3 = 2^2+2$

$3\times4 = 3^2+3$

$4\times5 = 4^2+4$

$\vdots$

$10\times11 = 10^2+10$

$\Rightarrow 1^2+2^2+3^2+\cdots+10^2+(1+2+3+\cdots10)$

$\Rightarrow \dfrac{10\times11\times21}{6} + 55$

$\Rightarrow 385 + 55$

$\Rightarrow 440$ Ans

Q. $\dfrac{1}{1\times2} + \dfrac{1}{2\times3} + \cdots + \dfrac{1}{10\times11}$

a, b, c

A $\dfrac{1}{1} - \cancel{\dfrac{1}{2}} + \cancel{\dfrac{1}{2}} - \dfrac{1}{3} + \cdots + \cancel{\dfrac{1}{10}} - \dfrac{1}{11}$

$1 - \dfrac{1}{11} = \dfrac{10}{11}$

or $\boxed{\dfrac{1}{b-a}\left(\dfrac{1}{a} - \dfrac{1}{c}\right)}$

$\dfrac{1}{1\times3} + \dfrac{1}{3\times5} + \dfrac{1}{5\times7} + \cdots + \dfrac{1}{11\times13}$

a, b, c

$= \dfrac{1}{2}\left[1 - \dfrac{1}{13}\right]$

$= \dfrac{6}{13}$ Ans

(*) $\dfrac{1}{2} + \dfrac{1}{6} + \dfrac{1}{12} + \cdots + \dfrac{1}{380}$

$= \dfrac{1}{1\times2} + \dfrac{1}{2\times3} + \dfrac{1}{3\times4} + \cdots + \dfrac{1}{19\times20}$

$= \dfrac{1}{1} - \dfrac{1}{2} + \dfrac{1}{2} - \dfrac{1}{3} \cdots + \dfrac{1}{19} - \dfrac{1}{20}$

a, b, d

$= \dfrac{1}{(b-a)}\left[\dfrac{1}{a} - \dfrac{1}{d}\right]$

$= \dfrac{1}{1}\left[1 - \dfrac{1}{20}\right] = \dfrac{19}{20}$ Ans

Q $\dfrac{1}{2^2-1} + \dfrac{1}{4^2-1} + \dfrac{1}{6^2-1} + \cdots + \dfrac{1}{20^2-1}$

$= \dfrac{1}{3} + \dfrac{1}{15} + \dfrac{1}{35} + \cdots + \dfrac{1}{399}$

$\dfrac{1}{19\times21}$

$= \dfrac{1}{2}\left(\dfrac{1}{1} - \dfrac{1}{21}\right)$

$= \dfrac{1}{2}\,\dfrac{20}{21} = \dfrac{10}{21}$ Ans

Divisibility Test :-

(i) Divided by 2 :- Last digit 0, 2, 4, 6, 8

(ii) Divisible by 3 :- Sum of digit divisible by 3.

(iv) Divisible by 4 :- Last two digit divisible by 4.

(v) divisible by 5 :- Last digit 0, 5.

(vi) divisible by 6 :- ~~Last~~ divisible by 2, 3 both.

(vii) divisible by 8 :- Last three digit divisible by 8

(viii) divisible by 9 :- Sum of digit divisible by 9

(ix) divisible by 11 :- diff of alternate sum is divisible by 11.

17821573 = 17 − 17 = 0

(x) Divisible by 12 :- A no. which is divisible by both 3 and 4.

$$a \overline{)\, N \text{ (divident)}}\ \ 2 \rightarrow \text{quotient}$$

divisor = a, r → remainder

$$\boxed{N = aq + r}$$

Q How many no. upto three digit is divisible by 19?

A 19) 999 (52
95
49
38

52 Ans

Note :-

(i) $a \overline{)\,N} \rightarrow r$; $a \overline{)\,2N} \rightarrow 2r$

$a \overline{)\,N} \rightarrow r$; $a \overline{)\,N^2} \rightarrow r^2$

(ii) $a \overline{)\,N_1} \rightarrow r_1$; $a \overline{)\,N_2} \rightarrow r_1$

$a \overline{)\,N_1 - N_2} \rightarrow 0 \ (r_1 - r_1) = 0$

Q. When 2272 and 875 is divided by a three digit no then their remaining is same. calculate No.

A Formula.

$a \overline{)\,N_1} \rightarrow r$; $a \overline{)\,N_2} \rightarrow r$

$a \overline{)\,N_1 - N_2} \rightarrow r = 0$

(ii) ⇒ $a \overline{)\,2272 - 875} = \overline{)\,1397}$

11) 1397 (127
11
29
22
77

Ans 127 Ans

Note :-

(1)

$$\frac{a^n}{a+1} \text{ then } r \begin{cases} \boxed{1} \Rightarrow n = \text{even} \\ \boxed{a} \Rightarrow n = \text{odd} \end{cases}$$

Eg:-

$$\frac{3^4}{4} = \frac{81}{4} = \boxed{r=1}$$

(ii)
$$\frac{27^{172}+1}{28} = \frac{1+1}{28} = 1+1 = 2$$

(iii)
$$\frac{27^{172}+27}{28} = \frac{27^{172}}{28} + \frac{27}{28}$$
$$= 1+27 = 28 \Rightarrow \underline{0}$$
$$28 \overline{)28} \quad r=0$$

(iv)
$$\frac{27^{172}-2}{28} = 1-2 = -1 \text{ (can't be)}$$
$$= 27 \text{ Ans}$$

(v)
$$\frac{2^{33}}{5} = \frac{2^{32}\times 2}{5}$$
$$= \frac{(2^2)^{16}\times 2}{5} = \frac{4^{16}\times 2}{5}$$
$$= 1\times 2 = 2 \text{ Ans}$$

(2)
$$\boxed{\frac{a^n}{a-1} \Rightarrow R = \boxed{1} \text{ always}}$$

Eg:-
$$\frac{17^{200}}{16} \quad r=1$$

$$\frac{127^{17}+1}{126} = 1+1 = 2$$

Q* $2^{96}-1$, is divisible by no. lying between 60 & 70. calculate sum of these no.

Ans:-
$$\frac{(64)^{16}-1}{63} \quad r=0$$

$$\frac{64^{16}-1}{65} \Rightarrow 1-1 = 0$$

No, $63, 65 = 128$ Ans

(iii)
$$\frac{x-a}{x-a} = R = 0$$

$$\boxed{\frac{x^n - a^n}{x-a} \Rightarrow R = 0, \; N = \text{Natural No}}$$

Eg:-
$$\frac{375^{176}-75^{176}}{375-75} = \frac{375^{176}-75^{176}}{300}$$
$$r = 0$$

(iv)
$$\boxed{\frac{x^n - a^n}{x+a} \Rightarrow r = 0, \; N = \text{even}}$$

Eg:-
$$\frac{375^{172}-125^{172}}{500} \quad r=0$$

$$\frac{375^{172}-125^{172}+1}{500} \Rightarrow r = 0+1 = 1$$

(v)
$$\boxed{\frac{x^n + a^n}{x+a} \Rightarrow r = 0 \text{ when } n = \text{odd}}$$

$$\frac{375^{171}+125^{171}}{500} \quad r=0$$

Q $10^n - 1$ is always divisible by 9 then n is.

A
$$\frac{10^n - 1^n}{9} \Rightarrow \text{any Natural No Ans}$$

Successive Division :-

Q Divide 37 successively by 5 & 3. cal rem.

Ans:-

$$5\overline{)37}\ (7 \quad 35 \quad 2 \qquad 3\overline{)7}\ (2 \quad 6 \quad 1$$

(2, 1)

Q When a no. is successively divided by 3 & 5 their remaining are 2 & 1 respectively. If order of division is interchanged then their remainder will be

Ans:-

3 ⊗ 5 ⊕
⊕ 2 1

$N = (5+1)\times 3 + 2$

$= 18 + 2 = 20$

$$5\overline{)20}\ (4 \quad 20 \quad 0 \qquad 3\overline{)4}\ (1 \quad 3 \quad 1$$

(0, 1) Ans

④ When same but 3 digit No.

5 3 6
3 2 1

$= \{(6+1)\times 3 + 2\}\times 5 + 3$

$= 118$

$$6\overline{)118}\ (19 \quad 6 \quad 58 \quad 54 \quad 4$$

$$3\overline{)118} \quad 9 \quad 28 \quad 27 \quad 1$$

$$5\overline{)118} \quad 115 \quad 3$$

Q In 1 to 1000 digit 2 appears how many times?

A $\Rightarrow 20\times 10 + 100$

100 ↓ (200−300)

$\Rightarrow 300$

Q 1 to 500, how many times digit 6 appears

A $20\times 5 = 100$

Q When sum of two no. is multiplied with these no. separately then their product are 161 and 200. respectively calculate No.

$a(a+b) = 161$

$b(a+b) = 200$

$(a+b)(a+b) = 361$

$(a+b)^2 = 361$

$a+b = \sqrt{361} = 19$

Now, $19a = 161$

$a = \frac{161}{19}$

$b = \frac{200}{19}$

Trick :-

Smallest No $= \frac{161}{\sqrt{200+161}}$

Larger no $= \frac{200}{\sqrt{200+161}}$

sum of No. $= \frac{200+161}{\sqrt{200+161}}$

diff $= \frac{200-161}{\sqrt{200+161}}$

Imp

Q student of a class are arranged in rows. If 4 more students are in each row then 2 less row are required. If 4 less row are required ~~if 4 less~~ students are in each row then 4 more are required. calculate

(i) No of students in each row

(ii) No of row

(iii) Total No of students

Ans

$$\boxed{\text{No of student} = \frac{S_1 S_2 (R_1 + R_2)}{S_1 R_2 - S_2 R_1}}$$

$$\boxed{\text{No of rows} = \frac{R_1 R_2 (S_1 + S_2)}{S_1 R_2 - S_2 R_1}}$$

$$\text{No of student in each row} = \frac{4\times4\ (2+4)}{4\times4 - 4\times2}$$

$$= \frac{\cancel{16}^{2} \times 6}{\cancel{8}} = 12 \text{ Ans}$$

$$\text{No of rows} = \frac{2\times4\ (8)}{\cancel{8}} = 8$$

$$\text{Total No of student} = 12\times8 = 96 \text{ Ans}$$

Any two digit No:-

$10x + y$ ↓ two digit

$100x + 10y + z$ ↓ Three digit

Sum:-

$72 = 7+2 = 9$

$\begin{array}{r} 72 \\ 27 \\ \hline 99 \end{array}$ $\quad 11\overline{)99}^{\,9}$

$\begin{array}{r} 85 \\ 58 \\ \hline 143 \end{array}$ $8+5$ $\quad 11\overline{)143}^{\,13}$

divisible by 11.

Difference:-

$\begin{array}{r} 72 \\ -27 \\ \hline 9)45(5 \end{array}$ $7-2$

$\begin{array}{r} 63 \\ 36 \\ \hline 9)27(3 \end{array}$ $6-3 = 3$

Q $\frac{1}{5} + 999\frac{494}{495} \times 99$

$\frac{1}{5} + \left(999 + \frac{494}{495}\right) \times 99$

$\frac{1}{5} + 999\times99 + \frac{494\times\cancel{99}}{\cancel{495}\,5}$

$\Rightarrow (1000-1)99 + \frac{494+1}{5}$

$\Rightarrow 99000 - \cancel{99} + \frac{\cancel{495}}{\cancel{5}}\cancel{99}$

$= 99{,}000$ Ans

Q $\frac{1}{7} + 99\frac{692}{693} \times 99 = ?$

9900 Ans

$\frac{1}{7} + \left(99 \times 99 + \frac{\cancel{692}}{\cancel{693}\ 7}\times\cancel{99}\right) \Rightarrow 9900$

$\Rightarrow 99\times99 + \frac{\cancel{693}}{\cancel{7}} \Rightarrow 99\times99 + 99$

$(100-1)99 + 99$

Q $99\frac{98}{99} \times 99 = ?$

Ans– $\left(99 + 1 - \frac{1}{99}\right) \times 99$

$\Rightarrow \left(100 - \frac{1}{99}\right) \times 99$

$\Rightarrow 9900 - \frac{99}{99} \Rightarrow 9900 - 1$

$\Rightarrow \underline{9899}$ A

Q $\dfrac{1}{x + \dfrac{1}{y + \dfrac{1}{2}}} = \dfrac{7}{37}$ then $x+y+z = ?$

A Reverse it

$x + \dfrac{1}{y + \frac{1}{2}} = \dfrac{37}{7}$

$x + \dfrac{1}{y + \frac{1}{2}} = 5 + \dfrac{2}{7}$

$\underline{x = 5}$ No $\dfrac{1}{y + \frac{1}{2}} = \dfrac{2}{7}$

$\Rightarrow y + \frac{1}{2} = \frac{7}{2}$

$= 3 + \frac{1}{2}$

$\left.\begin{array}{l} y = 3 \\ z = 2 \\ x = 5 \end{array}\right\}$ 10 A

Q $2^{40}, 5^{30}, 4^{20}, 6^{10}$ (4, 3, 2, 1)

$\Rightarrow 2^4, 5^3, 4^2, 6^1$

$\Rightarrow 16 \quad 125 \quad 16 \quad 6$

Q $\frac{1}{3} = 0.3333$

$= 0.\overline{3}$

$= \frac{3}{9}$

$0.\overline{12} = \frac{12}{99}$

$0.3\overline{21} = \frac{321-3}{990} = \frac{318}{99}$

$0.32\overline{1} = \frac{321-32}{900} = \frac{289}{900}$

Q $0.\overline{3} + 0.\overline{4} + 0.\overline{5} + \dots + 0.\overline{8}$

$\frac{3}{9} + \frac{4}{9} + \frac{5}{9} + \dots + \frac{8}{9}$

$\Rightarrow \frac{1}{9}[3+4+5+\dots+8]$ $\quad \frac{8 \times 9}{2} = 36 - 3 = 33$

$= \frac{1}{9}[33]$

$= 3 + \frac{6}{9}$

$= 3.\overline{6}$ A

Q. $\sqrt{4+2\sqrt{3}}$

$(a+b)^2 = a^2+b^2+2ab$

$(a-b)^2 = a^2+b^2-2ab$

(i) $\sqrt{4+2\sqrt{3}} \Rightarrow \sqrt{(\sqrt{2}+\sqrt{3})^2}$

$= \sqrt{2}+\sqrt{3}$

(ii) $\sqrt{8+2\sqrt{12}} = \sqrt{6}+\sqrt{2}$

$\sqrt{6}\ \ \sqrt{2}$

$\sqrt{8-2\sqrt{12}} = \sqrt{6}-\sqrt{2}$

(iii) $\sqrt{7+2\sqrt{12}} = \sqrt{4}+\sqrt{3}$

(iv) $\sqrt{3+\sqrt{5}} = ?$

$= \sqrt{\frac{2(3+\sqrt{5})}{2}}$

$= \sqrt{\frac{6+2\sqrt{5}}{2}} = \frac{\sqrt{5}+\sqrt{1}}{\sqrt{2}}$

Q If $x = 7+2\sqrt{12}$ then $\sqrt{x}+\frac{1}{\sqrt{x}}$

$x = \sqrt{4}+\sqrt{3}+2\sqrt{4}\times\sqrt{3}$

$= (\sqrt{4}+\sqrt{3})^2$

$\sqrt{x} = (\sqrt{4}+\sqrt{3})$

$\frac{1}{\sqrt{x}} = \sqrt{4}-\sqrt{3}$

$\sqrt{x}+\frac{1}{\sqrt{x}} = 4$ Ans

Q $a = \frac{\sqrt{3}}{2}$ then $\sqrt{1+a}+\sqrt{1-a} = ?$

$1+a = 1+\frac{\sqrt{3}}{2}$

$= \frac{(2+\sqrt{3})\times 2}{2\times 2} = \left(\frac{4+2\sqrt{3}}{4}\right)$

$= \frac{(\sqrt{3}+1)^2}{4}$

$\sqrt{1+a} = \frac{(\sqrt{3}+1)}{2}$

$\sqrt{1-a} = \frac{\sqrt{3}-1}{2}$

$\sqrt{1+a}+\sqrt{1-a} = \frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}$

$= \sqrt{3}$ Ans

Q $\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\cdots+\frac{1}{\sqrt{120}+\sqrt{121}}$

$\Downarrow$

$\frac{1}{1+\sqrt{2}}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}+\frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2}+\cdots$

$= \cancel{\sqrt{2}}-1+\cancel{\sqrt{3}}-\cancel{\sqrt{2}}+\cdots+\sqrt{121}-\cancel{\sqrt{120}}$

$= -1+11 = 10$ Ans

Some Tricks :-

$$\sqrt{a\sqrt{a\sqrt{a\sqrt{a\cdots}}}} = a$$

$$\sqrt{a\sqrt{a\sqrt{a\sqrt{a\cdots n}}}} = a^{\frac{2^n-1}{2^n}}$$

$$\sqrt{a+\sqrt{a+\sqrt{a+\cdots\infty}}} = \text{Big factor}$$

factor

$$\sqrt{a-\sqrt{a-\sqrt{a-\sqrt{a\cdots\infty}}}} = \text{small factor}$$

Eg:-

$$\sqrt{6+\sqrt{6+\sqrt{6\cdots}}} = 3$$

3×2

$$\sqrt{6-\sqrt{6-\sqrt{6-\sqrt{6}}}} = 2 \text{ A}$$

3×2

Note:-

$$\sqrt{a-\sqrt{a+\sqrt{a-\sqrt{a\cdots\infty}}}} =$$

$$\boxed{\frac{\sqrt{4a-3}+1}{2}}$$

Componendo dividendo :-

$$\frac{a}{b} = \frac{c}{d}$$

$$\boxed{\frac{a+b}{a-b} = \frac{c+d}{c-d}}$$

or, $\frac{a-1}{a+1} = \frac{c}{d}$

$$\Rightarrow \left[\frac{\cancel{2}a}{-2} = \frac{c+d}{c-d}\right]$$

Q $\frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}} = 3$ then $x = ?$

A

$$\frac{\cancel{2}\sqrt{x+1}}{\cancel{2}\sqrt{x-1}} = \frac{3+1}{3-1}$$

$$\Rightarrow \frac{x+1}{x-1} = \frac{16}{4} = 4$$

$$\Rightarrow \frac{2x}{2} = \frac{5}{3} \text{ Ans}$$

Q $7^{3.2}\times56^{3.1}\times8^{1.1}\times7^{1.1}\times8^{3.2} = 56^x$

$$(7\times8)^{3.2}\times56^{3.1}\times(7\times8)^{1.1} = 56^x$$

$$\Rightarrow 56^{3.2}\times56^{3.1}\times56^{1.1} = 56^x$$

$$\Rightarrow 56^{7.3} = 56^x$$

$$\Rightarrow \boxed{x = 7.4} \text{ Ans}$$

—*—*—

Q $10^{3x} = 125$ then 10^{-4x}

$$10^{3x} = 5^3$$

$$10^{\frac{3x}{4}} =$$

$$10^{3x\times\frac{-4}{3}} = 5^{3\times\frac{-4}{3}}$$

$$\Rightarrow 10^{-4x} = 5^{-4} = \frac{1}{5^4} \text{ Ans}$$

Q

$$a^{m^n} = (a^m)^n$$

$$m^n = mn$$

$$m^{n-1} = n \Rightarrow \boxed{m = n^{\frac{1}{n-1}}}$$

Ans

Q $2^x = 3^y = 6^{-2}$

then, $\frac{1}{x} + \frac{1}{y} + \frac{1}{2} = ?$

A

$2^x = 3^y = 6^{-2} = K$

$2 = K^{1/x}$

$3 = K^{1/y}$

$6 = x^{-1/2}$

$\Rightarrow \quad 3 \times 2 = 6$

$\Rightarrow \quad \frac{1}{x} + \frac{1}{y} = -\frac{1}{2} \Rightarrow \boxed{\frac{1}{x} + \frac{1}{y} + \frac{1}{2} = 0}$

Number system:-

Remainder Theorem:-

(i) $(a+b)^2 = a^2 + 2ab + b^2$

$\frac{(a+b)^2}{a}$

$\hookrightarrow \frac{(a+b)^2}{a} = \frac{b^2}{a} \rightarrow$ Remainder

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

remainder

$\frac{(a+b)^3}{a} = \frac{b^3}{a}$

Note:- $\boxed{\frac{(a+b)^n}{a} \Rightarrow \boxed{r = \frac{b^n}{a}}}$

(ii) $\frac{(a-b)^n}{a} = \frac{(-b)^n}{a}$

$\downarrow$

find Reminder

eg: $\frac{7^4}{6} = \frac{(6+1)^4}{6}$

$= \frac{1^4}{6}$

$= 1$

(ii) $\frac{8^4}{6} = \frac{(1+2)^4}{6}$

$= \frac{2^4}{6}$

$= \frac{64}{6} \Rightarrow$

$r = 4$

(iii) $\frac{7^4}{8} = \frac{(8-1)^4}{8}$

$= \frac{(-1)^4}{8}$

$= 1$

(iv) $\frac{9^4}{7} = \frac{(7-2)^4}{7} = \frac{(-2)^4}{7}$

$= \frac{16}{7}$

$\boxed{r = 2}$

Imp. $\frac{6^{35}}{35} = \frac{6\,(36)^{17}}{35}$

$\frac{8^4}{6} =$

$\Rightarrow \frac{6^{34} \cdot 6}{35}$

$\Rightarrow \frac{36^{17} \cdot 6}{35}$

$\Rightarrow 1 \times 6 = 6$ Ans

Q. $(432)^{217!} \times (23)^{141!}$

$= (2)^1 \times 3^1 = 6 \times 1 = 6 =$

* Combined divisibility Rule of 7, 11, 13 (Prime No)

659 624

diff = 35 — 7, 11, 13

से divisible होगा तब divisible होगा

$x \overline{) m (}$, r_1 $\quad x\overline{) n (}$, r_2 $\quad x\overline{) m+n (}$, r_3

$$\boxed{x = r_1 + r_2 - r_3}$$

* No of zero in $126! - 125!$

$125!(126-1)$

$125!(125)$

5	125
5	25
5	5
	1

$31 + (5\times5\times5) = 34$

Q. $1^2 \times 2^2 \times 3^3 \times 4^4 \ldots 100^{100} = 1300$ zeros

---

Q $\dfrac{4^{5684}}{6} = 4$ Remainder

$\dfrac{4^1}{6} = 4$

$\dfrac{4^2}{6} = 4$

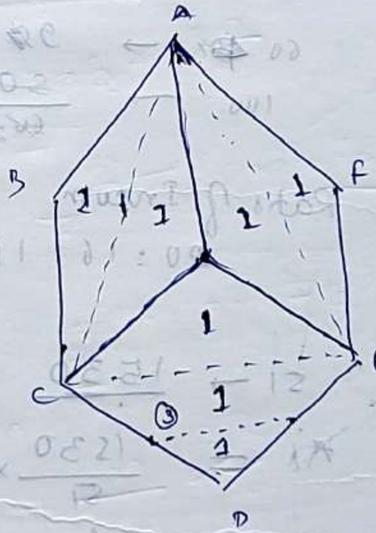
Polygon

1) $\boxed{r = \frac{a}{2}\cot\left(\frac{\pi}{n}\right)}$

2) $\boxed{R = \frac{a}{2}\operatorname{cosec}\left(\pi/n\right)}$

3) $\boxed{A = \frac{a^2}{4}\cot\left(\pi/n\right)}$

4) $\boxed{\frac{r}{R} = \cos\left(\pi/n\right)}$



Octagon:-

$\boxed{A = 2(2+\sqrt{2})\,a^2}$

$\cot 22\frac{1}{2} = 1+\sqrt{2}$

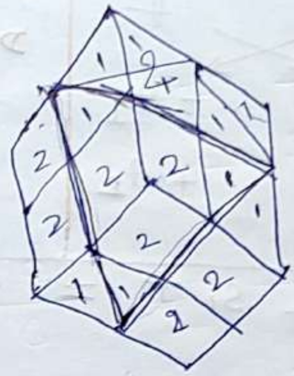
$\tan 22\frac{1}{2} = \sqrt{2}-1$

No. of sides $= \frac{360}{\theta_{ext}}$

No of diagonal $= \frac{n(n-3)}{2}$

$\boxed{(2n-4)\times 90 = \text{sum of Internal Angles}}$

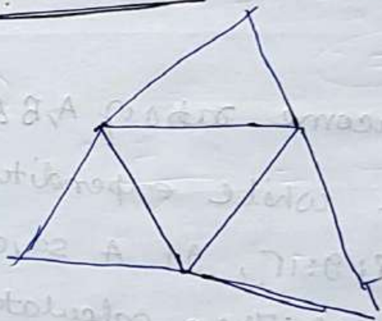
Question II



(8:3)

9:15

Tetrahedron :-



$\boxed{V = \frac{\sqrt{2}}{12}a^3}$

Continued proportion:-

$\Rightarrow a, b, c$

$a : c \Rightarrow a^2 : b^2$

$\Rightarrow a, b, c, d$

$a : d \Rightarrow a^3 : b^3$

$\Rightarrow a, b, c, d, \ldots n$

$\boxed{a : n = a^{n-1} : b^{n-1}}$

Some Important & Difficult Questions

Question:-

$S \quad t$

$+S_1 \quad -t_1$

$-S_2 \quad +t_2$

II nd method

$D_1 = D_2$

$$\boxed{\begin{aligned} -S_1 t_1 + S_1 t &= S_1 t_1 \\ S t_2 - S_2 t &= S_2 t_2 \end{aligned}}$$

Solve

Q The income ratio of A, B & C is 7 : 9 : 12 while expenditure are in 8 : 9 : 15, If A saves 25% of his income, Then calculate their saving ratio:-

A I → $7 \times 8 \times 4$ | $9 \times 8 \times 4$ | $12 \times 8 \times 4$

E → $8 \times 7 \times 3$ | $9 \times 7 \times 3$ | $15 \times 7 \times 3$

saving → 56 : 99 : 69

Now

I 100

S → 25

E = 75

$\frac{I}{E} = \frac{100}{75} = \frac{4}{3}$

Q The ratio of the expenditure of A, B and C are 16 : 12 : 9 respectively, and their saving are 20%, 25% and 40% respectively of their income. If the sum of their income is 1530, find B's, C's income

A E = 16 : 12 : 9

saving = 20%, 25%, 40%

Exp = 80% 75% 60%

A) 80 → 16
100 → 20

75% → 12
100% → 16

60% → 9
100 → $\frac{\cancel{90}}{\cancel{60}_2} \times 100 = \underline{15}$

Ratio of Income
20 : 16 : 15 ⇒ 51

51 → 1530

A's $= \frac{1530}{51} \times 20$

= 600 A

B's = $30 \times 16 = 480$

C's = $30 \times 15 = 450$ A